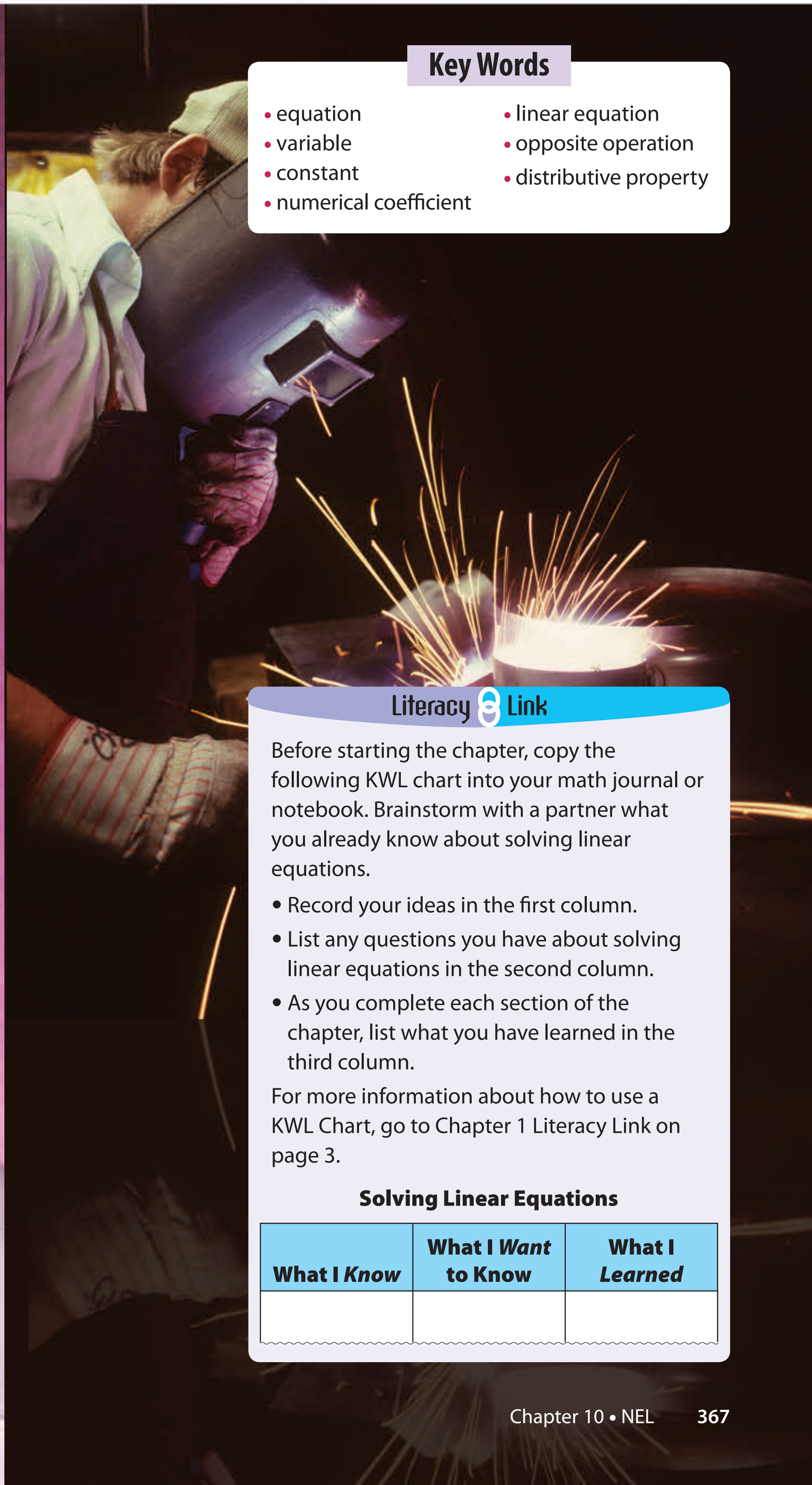
10

Solving Linear Equations

What do a scientist, accountant, meteorologist, professional athlete, and tradesperson have in common? All of these careers involve activities that can be modelled using linear equations. In fact, you would be amazed by how linear equations can represent so much of what goes on in the world around you.

What You Will Learn

- ❑ to use linear equations to model problems
- ❑ to solve problems involving linear equations



Key Words

- equation
- variable
- constant
- numerical coefficient
- linear equation
- opposite operation
- distributive property

Literacy Link

Before starting the chapter, copy the following KWL chart into your math journal or notebook. Brainstorm with a partner what you already know about solving linear equations.

- Record your ideas in the first column.
- List any questions you have about solving linear equations in the second column.
- As you complete each section of the chapter, list what you have learned in the third column.

For more information about how to use a KWL Chart, go to Chapter 1 Literacy Link on page 3.

Solving Linear Equations

What I *Know*	What I *Want* to Know	What I *Learned*

FOLDABLES™
Study Tool

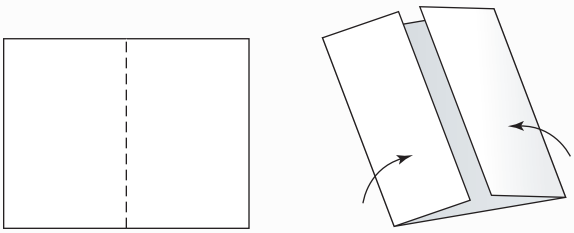
Making the Foldable

Materials

- 11 × 17 sheet of paper
- stapler
- scissors
- ruler

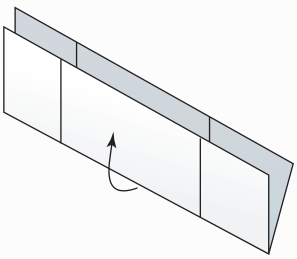
Step 1

Fold an 11 × 17 sheet of paper in half. Instead of creasing it, just pinch it at the midpoint. Fold the outer edges of the paper to meet at this midpoint.



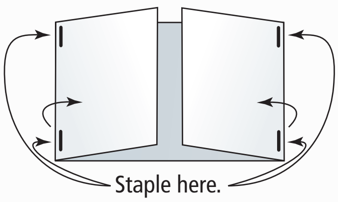
Step 2

Open the paper back up, and then fold it in half the other way so that the two horizontal edges meet.



Step 3

Fold the left and right ends along the creases toward the middle to make a large central pocket with one tab on the left and one on the right. Staple the tabs along the outside edge to hold the pocket together.

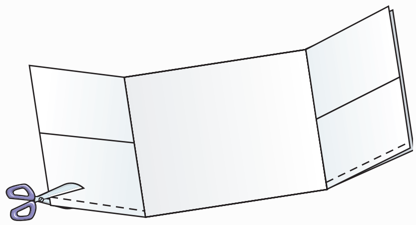


Step 4

Cut off the bottom crease of the left and right tabs as shown by the dashed lines in the visual with Step 5.

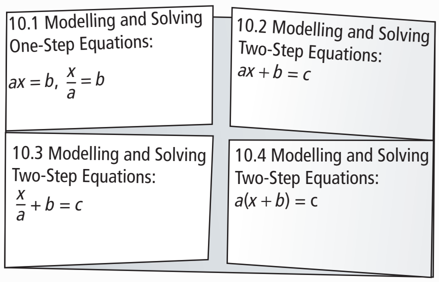
Step 5

Use a ruler to divide each tab in half horizontally, then cut along the lines to make two small booklets out of each tab, as shown below.



Step 6

Label the front of each small booklet as shown.



Using the Foldable

As you work through each section of Chapter 10, list and define the Key Words and record your notes about each example in the appropriate section of the Foldable.

In the large central pocket, store your work for the Math Link introduction on page 369 and the Math Links for each section. You may wish to place other examples of your work there as well. You can store your work on the Wrap It Up! in this pocket also.

On the back of the Foldable, make notes under the heading What I Need to Work On. Check off each item as you deal with it.

MATH LINK

Modelling Equations

Linear equations can be used to model everyday situations. You can even use your knowledge of linear equations to encrypt a password.

Jim's password is *weather*. He is going to encrypt this word using a two-step process.

Step 1: Jim assigns a number to represent each letter of the alphabet, as shown below.

1 = a	2 = b	3 = c	4 = d	5 = e	6 = f	7 = g
8 = h	9 = i	10 = j	11 = k	12 = l	13 = m	14 = n
15 = o	16 = p	17 = q	18 = r	19 = s	20 = t	21 = u
22 = v	23 = w	24 = x	25 = y	26 = z		

The number sequence for the password *weather* looks like 23 5 1 20 8 5 18.

Step 2: Jim uses the equation $y = 3x + 2$ to convert the number sequence to an encrypted number sequence. For example, the letter *w* was originally represented by the number 23. Substitute 23 into the equation:

$y = 3x + 2$
$y = 3(23) + 2$
$y = 71$

$3x$ means $3 \times x$.

The number 71 represents *w* in Jim's encrypted password.

a) What encrypted number sequence represents Jim's password of *weather*?

b) Discuss with a classmate how you might decrypt the encrypted number sequence 59 29 38 38 77. What process would you follow? What password does this number sequence represent?

c) Encrypt your own password using the values in the table above and a linear equation of your choice.

d) Exchange your password from part c) with your classmate. Decrypt each other's password and tell what equation was used to create the encryption system. If your classmate needs a hint, tell what number represents *e* in your encryption system.

Keep in mind that the most commonly used letters in the English language are, in order, E, T, A, O, I, N, S, H, R, D, and L.

10.1 Modelling and Solving One-Step Equations: $ax = b, \frac{x}{a} = b$

Focus on...

After this lesson, you will be able to...

- model problems with linear equations
- solve linear equations and show how you worked out the answer

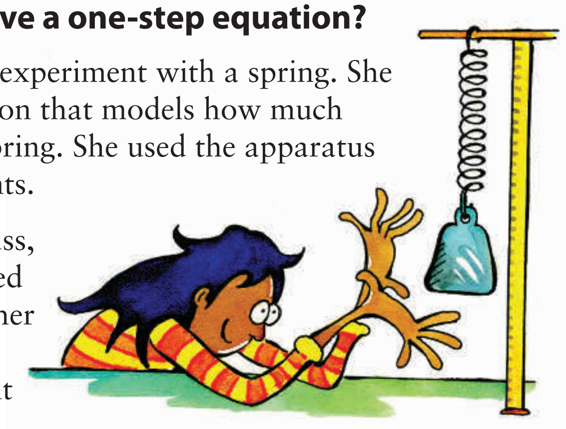
When Simone tried her new pair of Moon Shoes, she wondered what made them so bouncy. She discovered that they have springs inside that store energy. How do you think this energy is used to make the shoes bounce?

Explore the Math

Materials

- grid paper

How do you model and solve a one-step equation?



Simone decided to conduct an experiment with a spring. She wanted to determine an equation that models how much force is required to stretch a spring. She used the apparatus shown to take the measurements.

Every time Simone added a mass, the force on the spring increased and the spring stretched a further distance. The data that she collected during her experiment are shown in the table.

Trial	Force, F (newtons)	Distance Stretched, d (cm)
1	10	5
2	20	10
3	30	15
4	40	20
5	50	25

1. Draw a graph with Force on the horizontal axis and Distance Stretched on the vertical axis. Plot the values from the table.

2. a) How much more force is added for each trial?
   b) How much greater is the distance stretched each time force is added? Is the difference in the distance stretched the same for each consecutive trial?

3. What is the ratio, k, for the amount of force to the spring distance?

Literacy Link

An *equation* is a mathematical statement with two expressions that have the same value. The two expressions are separated by an equal sign. For example,

$2x = 3 \quad \frac{a}{3} = 5 \quad b = 4$

In the equation $4y - 7 = -3$,
- the numerical coefficient is 4
- the variable is y
- the constants are 7 and -3

Reflect on Your Findings

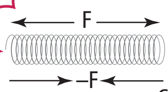
4. What is a **linear equation** that models the relationship between force and distance stretched?

The force, F, required to stretch a spring distance, d, can be modelled using the equation $F = kd$. Use the value of k that you determined in #3.

linear equation
- an equation that, when graphed, results in points that lie along a straight line
- examples are $y = 4x \quad d = \frac{c}{2}$ $5w + 1 = t$

5. a) If you use a force of 60 N, what is the distance the spring would stretch?
   b) How did you get your answer?

6. a) Imagine the spring is compressed instead of stretched. What would be the linear equation?
   b) How much force would it take to compress the spring 5 cm?



A positive force stretches a spring. A negative force compresses a spring.

Example 1: Solve an Equation

Solve each equation.

a) $3x = -12$ b) $\frac{r}{-2} = -7$

Solution

Method 1: Solve by Inspection

a) $3(-4) = -12$ or $\frac{-12}{3} = -4$

Ask yourself, "What number multiplied by 3 equals −12?"

Ask yourself, "What number results from dividing −12 by 3?"

The solution is $x = -4$.

b) $\frac{14}{-2} = -7$ or $-7 \times (-2) = 14$

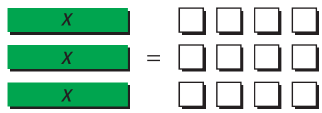
Ask yourself, "What number divided by −2 equals −7?"

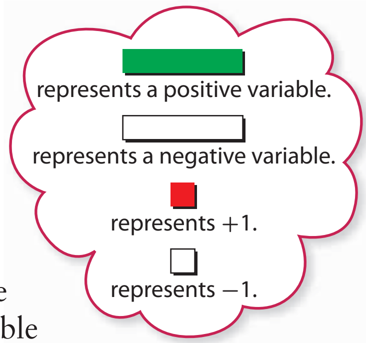
Ask yourself, "What number results from multiplying −7 by −2?"

The solution is $r = 14$.

Method 2: Solve Using Models and Diagrams

a) Use algebra tiles.

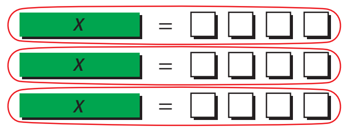




The three variable tiles represent $3x$.
The 12 negative 1-tiles represent -12.
The three variable tiles must have the same value as the 12 negative 1-tiles. Each variable tile must then have a value of four negative 1-tiles.

The solution is $x = -4$.

Check:

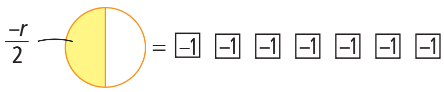


Left Side $= 3x$ Right Side $= -12$
$= 3(-4)$
$= -12$
Left Side = Right Side
The solution is correct.

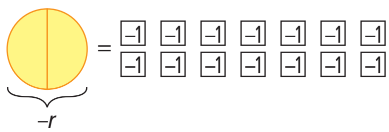
b) Use a diagram.
Let one whole circle represent $-r$.
Then, one half of the circle represents $\frac{-r}{2}$ or $-r \div 2$.

The expression $\frac{r}{-2}$ means the same as $\frac{-r}{2}$.

The seven white squares represent -7.



Since half the circle represents $\frac{-r}{2}$, you need to double the shading or multiply by two to represent $-r$. To balance the equation, you need to double or multiply by two the number of white squares.



There are now 14 white squares representing -14.
So, $-r = -14$.
The solution is $r = 14$.

What operation do you need to apply to both sides of the equation to solve for *r*?

Check:
Left Side $= \frac{r}{-2}$ Right Side $= -7$

$= \frac{14}{-2}$

$= -7$

Left Side = Right Side
The solution is correct.

Show You Know

Solve each equation. Check your answer.

a) $-3t = -36$ **b)** $\frac{n}{3} = -7$

Example 2: Divide to Apply the Opposite Operation

Simone uses a different spring in her experiment. The equation that models this new spring is $F = 12d$, where F is the force, in newtons, needed to stretch or compress the spring a distance, d, in centimetres. Simone applies a force of 84 N to compress the spring. What distance is the spring compressed?

Literacy Link

An *opposite operation* "undoes" another operation. Examples of opposite operations are

- subtraction and addition
- multiplication and division

You may sometimes hear opposite operations called *inverse operations*.

Solution

Since Simone compressed the spring, the force, F, is a negative number. Substitute -84 into the formula $F = 12d$. Then, isolate the variable to solve the equation.

$$
\begin{aligned}
F &= 12d \\
-84 &= 12d \\
\frac{-84}{12} &= \frac{12d}{12} \\
-7 &= d
\end{aligned}
$$

The opposite of multiplying by 12 is dividing by 12.

Literacy Link

Isolate the variable means to get the variable by itself on one side of the equation.

The spring was compressed a distance of 7 cm.

Check:

Left Side $= -84$ Right Side $= 12d$

$= 12(-7)$

$= -84$

Left Side = Right Side

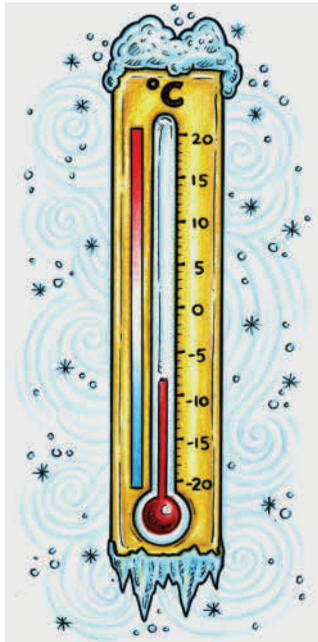
The solution is correct.

Show You Know

Solve by applying the opposite operation. Check your answer.

a) $-5b = -45$ **b)** $6f = -12$

Example 3: Multiply to Apply the Opposite Operation



For the month of January, the average afternoon temperature in Edmonton is $\frac{1}{3}$ the average afternoon temperature in Yellowknife. The average afternoon temperature in Edmonton is -8 °C. What is the average afternoon temperature in Yellowknife?

Solution

Let t represent the average afternoon temperature in Yellowknife.

The variable represents the unknown value.

The average afternoon temperature in Edmonton is $\frac{1}{3}$ the average afternoon temperature in Yellowknife, or $\frac{t}{3}$.

You can model the problem with the equation $\frac{t}{3} = -8$.

Solve the equation by applying the opposite operation.

Literacy Link

$\frac{1}{3}t$ is the same as $\frac{t}{3}$ or $t \div 3$.

$$\frac{t}{3} = -8$$
$$\frac{t}{3} \times 3 = -8 \times 3$$
$$t = -24$$

The opposite of dividing by 3 is multiplying by 3.

The average afternoon temperature in Yellowknife is -24 °C.

Check:

Left Side $= \frac{t}{3}$ $\quad$ Right Side $= -8$

$= \frac{-24}{3}$

$= -8$

The solution of -24 °C is correct.

Show You Know

Solve by applying the opposite operation. Check your answer.

a) $\frac{d}{-5} = 3$ $\quad$ **b)** $-6 = \frac{p}{7}$

Key Ideas

- There are several ways to solve equations involving integers.
  - Solve by inspection.

    $-2w = 6$

    $-2(-3) = 6$

    Ask yourself, "What number multiplied by −2 equals 6?"

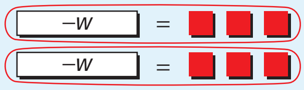
    or

    $\frac{6}{-2} = -3$

    Ask yourself, "What number results from dividing 6 by −2?"

    The solution is $w = -3$.
  - Model the equation using concrete materials and then balance it.

    $-2w = 6$

    Each negative variable tile must have a value of three positive 1-tiles.
    The positive variable tile must then have a value of three negative 1-tiles.
    The solution is $w = -3$.
  - Perform the opposite operation on both sides of the equal sign.

    $$\frac{w}{-2} = 6$$
    $$\frac{w}{-2} \times (-2) = 6 \times (-2)$$
    $$w = -12$$

    The opposite of dividing by −2 is multiplying by −2.

- Two methods you can use to check your solution are substitution and modelling:
  - Substitute your solution into the equation. Both sides should have the same value.

    $$\text{Left Side} = \frac{w}{-2} \qquad \text{Right Side} = 6$$
    $$= \frac{-12}{-2}$$
    $$= 6$$

    Left Side = Right Side

    The solution is correct.
  - Model the equation using concrete materials like algebra tiles as shown above.

Communicate the Ideas

1. Draw a diagram to show how you can model $\frac{x}{6} = -3$. Explain your diagram in words.

2. Give an example of an equation that has a variable with a negative integer value.

3. An unknown number is multiplied by 5. The result is -45.

a) Choose a variable. Write an equation to represent the situation.

b) Draw a picture to show how you might solve the equation.

4. Raj is solving the equation $\frac{n}{9} = -4$.

$$\frac{n}{9} = -4$$
$$\frac{n}{9} \times (-9) = -4 \times (-9)$$
$$n = 36$$

Is Raj's solution correct or incorrect? Explain.

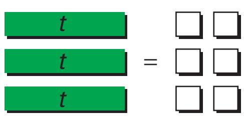
Check Your Understanding

Practise

For help with #5 to #10, refer to Example 1 on pages 371–372.

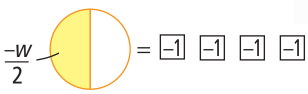
5. Write the equation modelled by each diagram.

a)

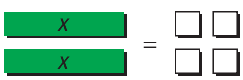


Recall that ■ represents +1 and □ represents −1.

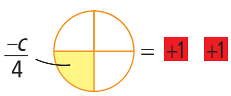
b)



c)

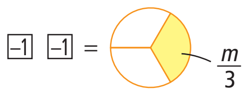


d)

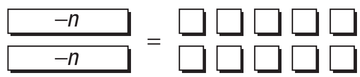


6. Write the equation represented by each model.

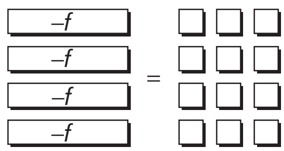
a)



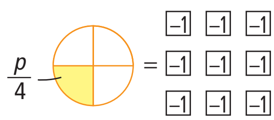
b)



c)



d)



7. Solve by inspection.

a) $-8j = 64$

b) $5n = -25$

c) $-6 = \frac{k}{3}$

d) $\frac{x}{-11} = -4$

8. Use mental math to solve each equation.

a) $-12 = 3r$

b) $-16 = -4p$

c) $-30 = \frac{t}{2}$

d) $\frac{d}{-4} = 5$

9. Use models or diagrams to solve each equation.

a) $2k = -8$

b) $-3 = \frac{t}{4}$

10. Solve each equation using models or diagrams.

a) $3b = -15$

b) $\frac{x}{-3} = -3$

For help with #11 to #14, refer to Example 2 on page 373.

11. By what number would you divide both sides of the equation to solve it?

a) $-3x = 9$

b) $-36 = -4g$

c) $72 = -9t$

d) $4p = -8$

12. By what number would you divide both sides of the equation to solve it?

a) $-10 = 5w$

b) $-48 = -4c$

c) $4y = -400$

d) $-84 = -21b$

13. Solve each equation using the opposite operation. Check your answer.

a) $4s = -12$

b) $-156 = -12j$

c) $-4j = 104$

d) $-108 = -27t$

14. Use the opposite operation to solve each equation. Verify your answer.

a) $8f = -56$

b) $-5q = 45$

c) $-2h = -42$

d) $14k = -70$

For help with #15 to #18, refer to Example 3 on page 374.

15. By what number would you multiply both sides of the equation to solve it?

a) $13 = \frac{g}{-6}$

b) $\frac{m}{3} = -25$

c) $-6 = \frac{n}{-21}$

d) $\frac{z}{17} = 6$

16. By what number would you multiply both sides of the equation to solve it?

a) $\frac{s}{11} = 9$

b) $-6 = \frac{y}{-12}$

c) $\frac{w}{4} = -13$

d) $16 = \frac{x}{-3}$

17. Solve each equation using the opposite operation. Check your answer.

a) $\frac{t}{3} = -12$

b) $12 = \frac{h}{-10}$

c) $\frac{s}{-7} = 15$

d) $-63 = \frac{x}{-9}$

18. Use the opposite operation to solve each equation. Verify your answer.

a) $\frac{y}{5} = -4$

b) $-6 = \frac{k}{-8}$

c) $-1 = \frac{b}{10}$

d) $\frac{r}{12} = 15$

Apply

19. Show whether $x = -2$ is the solution to each equation.

a) $-8x = 16$

b) $10x = -20$

c) $-5x = 10$

d) $36 = 18x$

20. Show whether $y = 12$ is the solution to each equation.

a) $3 = \frac{y}{-4}$

b) $\frac{y}{-36} = -3$

c) $2 = \frac{y}{24}$

d) $\frac{y}{-6} = -2$

21. For the month of January, the average afternoon temperature in Calgary is $\frac{1}{4}$ the average morning temperature. The average afternoon temperature is -4 °C. What is the average morning temperature?

a) If m represents the average morning temperature, what equation models this problem?

b) Solve the equation. Verify your answer.

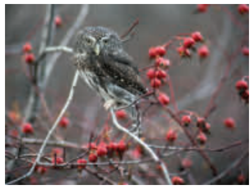
22. Nakasuk's snowmobile can travel 13 km on a litre of gas. He is going to visit his aunt in a community 312 km away. Nakasuk wants to know how many litres of gas he needs to travel to his aunt's community.

a) Write an equation in the form $ax = b$ to represent this problem. What does your variable represent?

b) How many litres of gas does Nakasuk need?

23. The height of a great grey owl is five times the height of a pygmy owl. A great grey owl can grow to 85 cm.

Great Grey Owl



Pygmy Owl

a) Model this problem with an equation of the form $ax = b$. Tell what your variable represents.

b) What is the height of the pygmy owl?

24. Lucy is making four pairs of mitts. She has 144 cm of trim to sew around the cuffs of the mitts. How much trim does she have for each mitt?

a) Write an equation to represent this situation.

b) Solve the equation.

25. People can be left-handed, right-handed, or ambidextrous. The number of boys in Canadian secondary schools who are left-handed is about $\frac{1}{7}$ of the number of boys who are right-handed. About 11% of boys are left-handed. Write and solve an equation to determine what percent of boys are right-handed.

Did You Know?

There are more ambidextrous students in Canada than there are left-handed students. *Ambidextrous* means that you are able to use your left hand and right hand with equal ability.

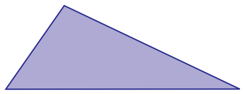
26. Kim works at an art gallery. An art dealer offers her a sculpture for $36 000. The dealer says the current value of the sculpture is twice its value the previous year.

a) What was its value the previous year?

b) If the sculpture's value increases at the same rate next year, what will the new value be?

Extend

27. The area of the triangle shown is 30 cm². Write and solve an equation to determine its height.



12 cm

28. Workers are repairing a section of road that is 5 km long. The speed limit has been changed from 50 km/h to 20 km/h. How many minutes does this add to the drive along this section of road?

29. The formulas that give the length of time for sound to travel underwater are

$t = \dfrac{d}{149\ 700}$ for fresh water, and

$t = \dfrac{d}{150\ 000}$ for salt water, where t is time, in seconds, and d is distance, in centimetres.

a) If a sound travels for 2 s, what distance does it travel in metres in fresh water? in salt water?

b) Two scientists are doing an underwater study of dolphin sounds. Sandra is 90 cm away from a freshwater dolphin. Donald is 1 m away from a saltwater dolphin. Who hears each sound in less time, Sandra or Donald? Show your work.

MATH LINK

Have you ever dropped Silly Putty® onto a hard surface? It bounces! The greater the height from which a ball of Silly Putty® is dropped, the higher it bounces.

a) Design and perform an experiment that allows you to record how high a ball of Silly Putty® bounces when dropped from different heights.

b) Determine an equation that models the results of your experiment. Write the equation in the form $b = kh$, where h is the height from which the Silly Putty® ball is dropped, b is the height of the first bounce, and k is a numerical coefficient that you will determine from your experiment.

WWW Web Link

For a Silly Putty® recipe, go to www.mathlinks8.ca and follow the links.

10.2 Modelling and Solving Two-Step Equations: $ax + b = c$

Focus on...

After this lesson, you will be able to...

- ❑ model problems with two-step linear equations
- ❑ solve two-step linear equations and show how you worked out the answer

Cali borrowed \$19 from her brother to purchase a CD. The next day, she paid back \$3. She will pay back the rest at a rate of \$4/week. Suggest ways that Cali might determine how long it will take to pay back her brother.

Explore the Math

How do you solve two-step equations of the form $ax + b = c$?

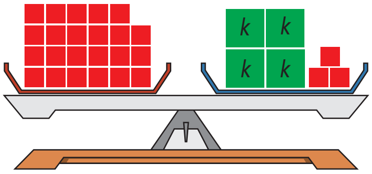
Example 1: Model With a Balance Scale

The city in Canada with the highest average wind speed is St. John's, Newfoundland. The city with the lowest average wind speed is Kelowna, British Columbia. The relationship between the wind speeds can be modelled using the equation $s = 4k + 3$, where s represents the wind speed in St. John's and k represents the wind speed in Kelowna. If the average wind speed in St. John's is 23 km/h, what is the average wind speed in Kelowna?

Solution

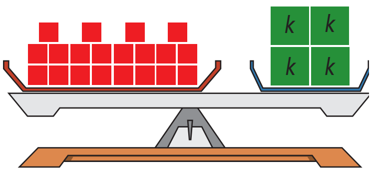
Substitute the known wind speed into the equation.
The wind speed for St. John's is 23 km/h.
$23 = 4k + 3$

You can model this equation using blocks and a scale.



To isolate the variable, first remove the three unit blocks from the right side of the scale. To keep the scale balanced, you must remove the same number of unit blocks from the left side of the scale.

There are four k blocks on the right side of the scale. There are 20 unit blocks on the left side of the scale. For the scale to balance, each k block must have a mass of five unit blocks.



The average wind speed in Kelowna is 5 km/h.

Check:

Left Side = 23 Right Side $= 4k + 3$
$= 4(5) + 3$
$= 20 + 3$
$= 23$

Left Side = Right Side

The solution is correct.

WWW Web Link

To practise solving linear equations using a balance scale, go to www.mathlinks8.ca and follow the links.

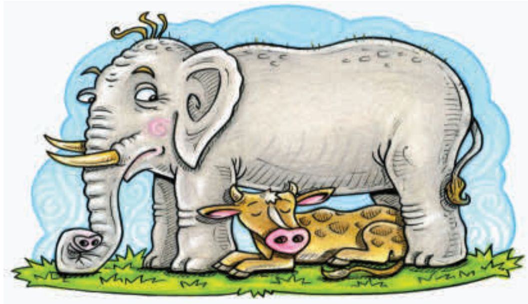
Show You Know

Solve each equation by drawing a diagram of a balance scale and blocks.

a) $6n + 6 = 12$ **b)** $13 = 9 + 2p$

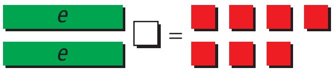
Example 2: Model With Algebra Tiles

A cow sleeps 7 h a day. This amount of sleep is 1 h less than twice the amount an elephant sleeps a day. How long does an elephant sleep?



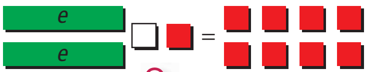
Solution

Let e represent the hours an elephant sleeps.
A cow sleeps 1 h less than twice what an elephant sleeps, or $2e - 1$.
A cow sleeps 7 h.
$2e - 1 = 7$



One less means you need to subtract 1, and *twice* means you need to multiply by 2.

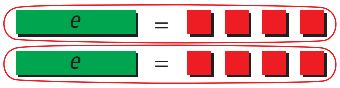
To isolate the variable, first add one positive 1-tile to both sides.



−1 +1
This is called a zero pair because $-1 + (+1) = 0$.

The negative 1-tile and positive 1-tile on the left side equal zero.

The two variable tiles must have the same value as the eight positive 1-tiles on the right side of the model. Each variable tile must then have a value of four positive 1-tiles.



An elephant sleeps 4 h a day.

Check:

Left Side $= 2e - 1$ Right Side $= 7$

$= 2(4) - 1$

$= 8 - 1$

$= 7$

Left Side = Right Side

The solution is correct.

Literacy Link

To solve a problem, you sometimes need to translate words into an equation. For example, *two more* means you need to add 2, and *three times* means you need to multiply by 3. What other words translate into math operations?

Literacy Link

Order of Operations

When substituting a value into the equation, make sure to use the correct order of operations:

- first, multiply and divide in order from left to right
- finally, add and subtract in order from left to right

Show You Know

Model each equation with algebra tiles. Then, solve.

a) $2g + 4 = -6$ **b)** $-2r - 7 = -11$

How do you represent $-2r$ using algebra tiles?

Example 3: Apply the Opposite Operations

Cali borrowed \$19 from her brother. The next day, she paid back \$3. To pay off the rest of the debt, she will give him \$4/week. How many weeks will it take her to pay off the debt?

> The amount Cali still needs to pay back is \$4 times the number of weeks, or "$4w$". The amount of \$3 that she has already paid back is represented by "+ 3."

Solution

Let w represent the number of weeks.
Cali is paying off \$4/week and has already paid \$3. The total she will pay is $4w + 3$. She owes a total of \$19.
$4w + 3 = 19$

> If you think of money owed as being negative, you can use the equation $-4w - 3 = -19$. When you solve it, the value of w is still the same.

Isolate the variable w to solve the equation.

$$4w + 3 = 19$$

$$4w + 3 - 3 = 19 - 3 \qquad \text{Subtract 3 from both sides of the equation.}$$

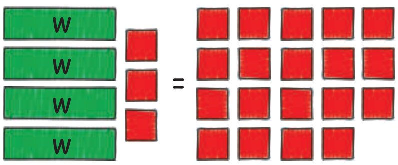
$$4w = 16$$

$$\frac{4w}{4} = \frac{16}{4} \qquad \text{Divide both sides of the equation by 4.}$$

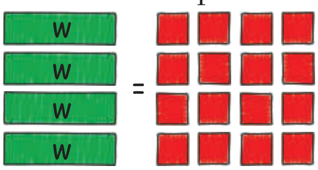
$$w = 4$$

It will take Cali four weeks to pay off her debt.

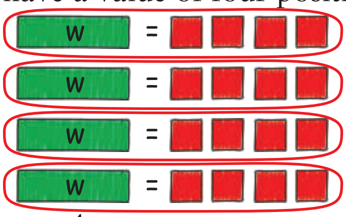
Check:



Subtract three positive 1-tiles from both sides.



The four variable tiles must have the same value as the 16 positive 1-tiles on the right side of the model. Each variable tile must then have a value of four positive 1-tiles.



$w = 4$

The solution is correct.

Show You Know

Solve by applying the opposite operations.

a) $4 + 26g = -48$ **b)** $-3x + 7 = 19$

Literacy Link

Reverse Order of Operations

When isolating a variable, follow the reverse order of operations:

- add and/or subtract
- multiply and/or divide

Strategies

Draw a Diagram

Key Ideas

$$
\begin{aligned}
2x - 4 &= 8 \\
2x - 4 + 4 &= 8 + 4 \\
2x &= 12 \\
\frac{2x}{2} &= \frac{12}{2} \\
x &= 6
\end{aligned}
$$

- To solve an equation, isolate the variable on one side of the equal sign. When undoing the operations performed on the variable, follow the reverse order of operations:
  - add and/or subtract
  - multiply and/or divide
- Two methods you can use to check your solution are substitution and drawing a diagram:
  - Substitute your answer into the equation. Both sides should have the same value.

    Left Side $= 2x - 4$ Right Side $= 8$

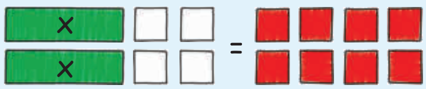
    $= 2(6) - 4$

    $= 12 - 4$

    $= 8$

    Left Side = Right Side

    The solution is correct.
  - Draw a diagram to model the equation.

    Add four positive 1-tiles to both sides.

    The four negative 1-tiles and the four positive 1-tiles on the left side equal zero. The two variable tiles must have the same value as the 12 positive 1-tiles. That means each variable tile must have a value of six positive 1-tiles .

    The solution of $x = 6$ is correct.

Communicate the Ideas

1. Draw diagrams to show how you would solve the equation $24 = 14 - 5x$ using algebra tiles. Explain each step in words.

2. a) Describe how you would isolate the variable in the equation $5x + 10 = 40$.

b) If the equation is changed to $5x - 10 = 40$, would you use the same process to isolate the variable? Explain.

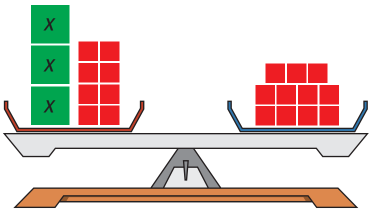
Check Your Understanding

Practise

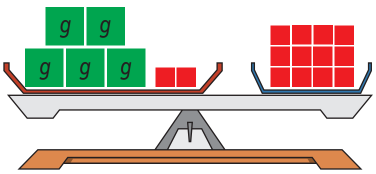
For help with #3 and #4, refer to Example 1 on page 380–381.

3. Solve the equation modelled by each balance scale. Check your solution.

a)

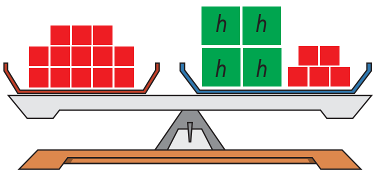


b)

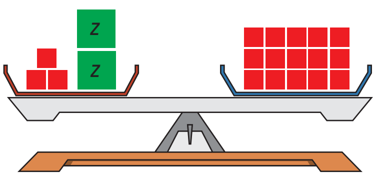


4. Solve the equation represented by each balance scale. Verify your solution.

a)



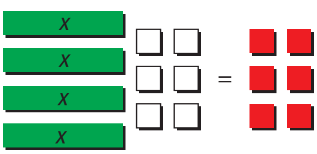
b)



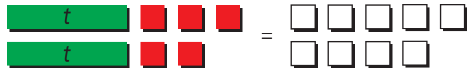
For help with #5 and #6, refer to Example 2 on page 382.

5. Solve each equation modelled by the algebra tiles. Check your solution.

a)

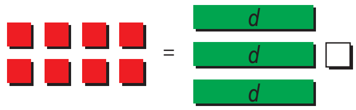


b)

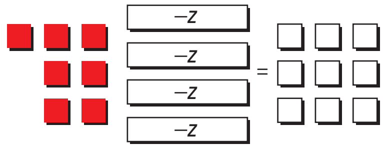


6. Solve each equation represented by the algebra tiles. Verify your solution.

a)



b)



For help with #7 to #10, refer to Example 3 on page 383.

7. What is the first operation you should perform to solve each equation?

a) $4r - 2 = 14$

b) $3 - 3x = -9$

c) $-22 = -10 + 2m$

d) $53 = -9k - 1$

8. What is the second operation you should perform to solve each equation in #7?

9. Solve each equation. Check your answer.

a) $6r + 6 = 18$

b) $4m + 8 = 12$

c) $39 + 9g = 75$

d) $-37 = 8f - 139$

10. Solve. Verify your answer.

a) $-17 = 3k + 4$

b) $29 = -14n + 1$

c) $8x - 7 = -31$

d) $-10 = 4n - 12$

Apply

11. Show whether $x = -3$ is the solution to each equation.

a) $-8x - 1 = 25$

b) $3 - 7x = -24$

c) $29 = -10x - 1$

d) $30 = 6x + 12$

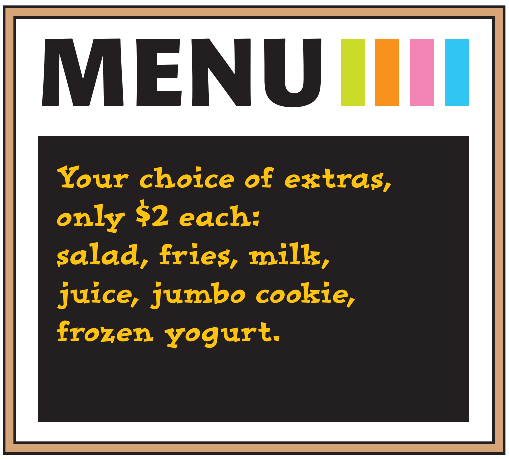
12. Matt is saving \$750 to buy a clothes dryer. If he triples the amount he has saved so far, he will have \$30 more than he needs. The situation can be modelled as $3s - 30 = 750$, where s represents the amount he has saved so far.

a) Explain how $3s - 30 = 750$ models the situation.

b) How much money has Matt saved so far?

c) What other strategy could you use to determine Matt's savings?

13. You are buying lunch at Sandwich Express. The cost is \$4 for a sandwich and \$2 each for your choice of extras. You have \$10. The equation to determine how many extras you can get is $10 = 2e + 4$, where e is the number of extras. How many extras can you buy if you spend all of your money?



14. The percent of elementary school students who choose hockey as their favourite physical activity is 14%. This percent of students is 2% more than four times the percent who choose skiing.

a) Let s represent the percent of students who choose skiing. What equation models this situation?

b) Solve the equation to find the percent of students who choose skiing.

15. If Jennifer doubled the money that she has in her account now and then took out \$50, she would have enough left in her account to buy a new bike that costs \$299. Write and solve an equation to determine how much money Jennifer has now.

16. A classroom's length is 3 m less than two times its width. The classroom has a length of 9 m. Write and solve an equation to determine the width of the classroom.

17. An eagle is hunting a bird in flight. The eagle begins its descent from a height of 74 m. The eagle reaches its prey at a height of 3 m. This situation can be modelled using the formula $74 = 3 + 6t$, where t represents the time in seconds.

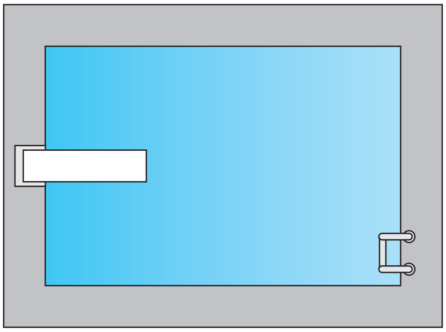
a) What do you think the value of 6 represents in the equation?

b) After how many seconds does the eagle reach its prey? Give your answer to the nearest tenth of a second.

Extend

18. The base of an isosceles triangle is 6 m less than two times one side. The base is 24 m. What is the area of the triangle?

19. The deck around a swimming pool has the same width all the way around. The perimeter of the pool is 50 m. The outside perimeter of the deck is 74 m. What is the width of the deck?



20. The variable m is a positive integer. The variable n is an integer from 0 to 9. Identify all of the values for m that would satisfy the equation $3m + n = 2008$.

21. Mallika walked at 2 km/h for 2 h and then cycled at x km/h for 3 h. If the average speed for the whole journey was 3 km/h, how fast did she cycle? Give your answer to the nearest tenth of a kilometre per hour.

MATH LINK

When any object falls, it picks up more and more speed as it falls. In fact, a falling object increases its speed by about 10 m/s for every second it falls.

Suppose a stone is dislodged from the side of a canyon and falls with an initial speed of 5 m/s. It hits the water below it at a speed of 45 m/s.

Write and solve an equation to determine the amount of time the stone fell before it hit the water.

10.3 Modelling and Solving Two-Step Equations: $\frac{x}{a} + b = c$

Focus on...

After this lesson, you will be able to...

- model problems with two-step linear equations
- solve two-step linear equations and show how you worked out the answer

Materials

- algebra tiles

The mass of a Persian cat is typically 2 kg less than $\frac{1}{3}$ of the average mass of a border collie. The average mass of a Persian cat is 4 kg. Describe how you might determine the average mass of a border collie.

Explore the Math

How do you model and solve two-step equations of the form $\frac{x}{a} + b = c$?

1. Use d to represent the average mass of a border collie. What is an equation that models the relationship between the masses of the border collie and the Persian cat?

2. How could you use a model or diagram to represent your equation?

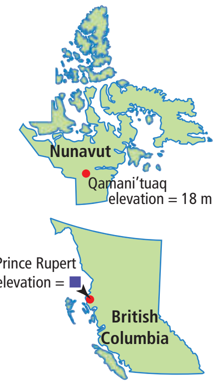
3. Use your model or diagram to help you solve this equation.
   a) What is the first thing you do to isolate d?
   b) What equation does your model or diagram represent now?
   c) What do you do next?
   d) What is the average mass of a border collie?

Reflect on Your Findings

4. a) Why is this type of equation called a two-step equation?
   b) How is solving an equation of the form $\frac{x}{a} + b = c$ similar to solving one of the form $ax + b = c$? How is it different?

Example 1: Model Equations

The elevation of Qamani'tuaq, Nunavut, is 1 m less than $\frac{1}{2}$ the elevation of Prince Rupert, British Columbia. If the elevation of Qamani'tuaq is 18 m, what is the elevation of Prince Rupert?



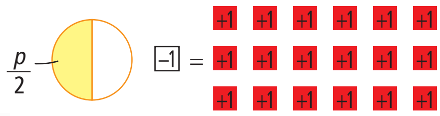
Did You Know?

The community of Qamani'tuaq, Nunavut, is also known as Baker Lake.

Solution

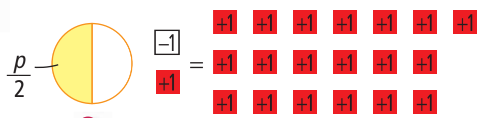
Let p represent the elevation of Prince Rupert.

The equation that models this situation is $\frac{p}{2} - 1 = 18$.



What part of the equation represents $\frac{1}{2}$ the elevation? What part of the equation represents 1 m less?

To isolate the variable, first add one red +1 square to both si des.



You need two equal parts to fill the circle.

The $\frac{1}{2}$ circle must have the same value as +19.
Multiply by 2 to fill the circle.
To balance the equation, multiply +19 by 2.
The variable p must then have a value of $2 \times 19 = 38$.
The elevation of Prince Rupert is 38 m.

Check:

Left Side $= \frac{p}{2} - 1$ Right Side $= 18$

$$= \frac{38}{2} - 1$$
$$= 19 - 1$$
$$= 18$$

Left Side = Right Side

The solution is correct.

Show You Know

Solve by modelling each equation.

a) $\frac{x}{4} - 5 = -7$ **b)** $\frac{-p}{3} + 1 = -4$

Example 2: Apply the Reverse Order of Operations

During the 2006–2007 NHL season, Kristian Huselius of the Calgary Flames had a total of 41 more than $\frac{1}{2}$ the number of shots on goal as Jarome Iginla. If Huselius had 173 shots on goal, how many did Iginla have?

Use the reverse order of operations. Add and subtract first, then multiply and divide.

Why would you not use models to solve this problem?

Solution

Let j represent the number of shots on goal Jarome Iginla had.

This situation can be modelled with the equation $\frac{j}{2} + 41 = 173$.

$$\frac{j}{2} + 41 - 41 = 173 - 41$$ Subtract 41 from both sides of the equation.

$$\frac{j}{2} = 132$$

$$\frac{j}{2} \times 2 = 132 \times 2$$ Multiply both sides of the equation by 2.

$$j = 264$$

Jarome Iginla had 264 shots on goal during the 2006–2007 season.

Check:

$$\begin{aligned} \text{Left Side} &= \frac{j}{2} + 41 \qquad \text{Right Side} = 173 \\ &= \frac{264}{2} + 41 \\ &= 132 + 41 \\ &= 173 \end{aligned}$$

Left Side = Right Side

The solution is correct.

Show You Know

Solve by applying the reverse order of operations.

a) $\frac{-x}{12} - 6 = 4$ **b)** $-4 = 3 + \frac{k}{7}$

Key Ideas

- To solve an equation, isolate the variable on one side of the equal sign. When undoing the operations performed on the variable, follow the reverse order of operations:
  - subtract and/or add
  - multiply and/or divide

$$
\begin{aligned}
\frac{x}{-4} + 3 &= 5 \\
\frac{x}{-4} + 3 - 3 &= 5 - 3 \\
\frac{x}{-4} &= 2 \\
\frac{x}{-4} \times (-4) &= 2 \times (-4) \\
x &= -8
\end{aligned}
$$

$$
\begin{aligned}
5 &= 2 - \frac{n}{4} \\
5 - 2 &= 2 - 2 - \frac{n}{4} \\
3 &= -\frac{n}{4} \\
3 \times 4 &= -\frac{n}{4} \times 4 \\
12 &= -n \\
12 \div (-1) &= -n \div (-1) \\
-12 &= n
\end{aligned}
$$

- One method you can use to check your answer is substituting it back into the equation. Both sides of the equation should have the same value.

$$
\begin{aligned}
\text{Left Side} &= \frac{x}{-4} + 3 \qquad \text{Right Side} = 5 \\
&= \frac{-8}{-4} + 3 \\
&= 2 + 3 \\
&= 5
\end{aligned}
$$

Left Side = Right Side

The solution is correct.

Communicate the Ideas

1. Describe a situation that can be modelled with the equation $\frac{x}{4} - 2 = 3$.

2. Describe how to isolate the variable when solving $12 - \frac{n}{5} = 6$. Compare your answer with a classmate's.

3. Manjit believes that the first step in solving the equation $\frac{x}{-4} + 7 = 9$ is to multiply both sides of the equation by -4 as shown.

$\frac{x}{-4} \times (-4) + 7 = 9 \times (-4)$

Is he correct? Explain.

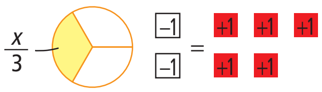
Check Your Understanding

Practise

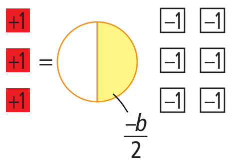
For help with #4 to #7, refer to Example 1 on page 389.

4. Solve the equation modelled by each diagram. Check your solution.

a)

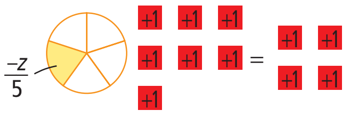


b)

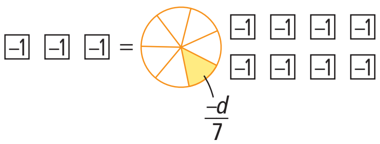


5. Solve the equation represented by each diagram. Verify your solution.

a)



b)



6. Draw a model for each equation. Then, solve. Verify your answer.

a) $-5 + \frac{g}{-2} = 3$ **b)** $-3 = 7 + \frac{n}{5}$

7. For each equation, draw a model. Then, solve. Check your answer.

a) $\frac{f}{-5} + 3 = -2$ **b)** $-1 = \frac{n}{8} - 4$

For help with #8 to #11, refer to Example 2 on page 390.

8. What is the first operation you should perform to solve each equation?

a) $\frac{t}{-5} + 12 = 9$ **b)** $\frac{p}{13} - 2 = -3$

c) $\frac{-k}{12} + 6 = 15$ **d)** $14 = 11 - \frac{x}{3}$

9. What is the second operation you should perform to solve each equation in #8?

10. Solve each equation. Verify your answer.

a) $2 + \frac{m}{3} = 18$ **b)** $\frac{c}{-8} - 8 = -12$

c) $16 = 9 + \frac{b}{-8}$ **d)** $-3 = \frac{n}{-7} + 19$

11. Solve. Check your answer.

a) $4 + \frac{j}{-8} = 8$ **b)** $\frac{r}{2} - 12 = -12$

c) $15 = -5 + \frac{x}{-6}$ **d)** $-2 = \frac{n}{13} - 17$

Apply

12. Show whether $n = -72$ is the solution to each equation.

a) $6 + \frac{n}{9} = 14$ **b)** $2 = 14 + \frac{n}{6}$

c) $\frac{n}{-3} + 6 = -18$ **d)** $-17 = \frac{n}{36} - 15$

13. The amount of sleep needed each night by people 18 years old or younger can be modelled by the equation $s = 12 - \frac{a}{4}$, where the amount of sleep in hours is s, and the age in years is a.

a) If 10 h is the amount of sleep Brian needs, how old is he likely to be?

b) Natasha is 13. She gets 8 h of sleep each night. Is this enough? Explain your reasoning.

14. The cost of a concert ticket for a student is \$2 less than one half of the cost for an adult. The cost of the student ticket is \$5. Let a represent the cost of an adult ticket. Write and solve an equation to determine the cost of an adult ticket.

15. In the following formula, T is the air temperature in degrees Celsius at an altitude of h metres, and t is the ground temperature in degrees Celsius:

$T = t - \frac{h}{150}$.

a) If the ground temperature is 25 °C, what is the temperature outside an aircraft at an altitude of 7500 m?

b) What is the altitude of the same plane if the outside air temperature is −35 °C?

16. In Canada, the percent of secondary school students who say their favourite subject is science is 1% less than $\frac{1}{2}$ of the number of students who choose math. The percent of students who prefer science is 6%. Write and solve an equation to determine what percent of students prefer math.

Extend

17. The recommended energy requirement per day for 14-year-old boys depends on how active they are. The requirement can be modelled by the following equations, where a is the age and C is the number of Calories.

Active	Moderately Active
$a = \frac{C}{100} - 17$	$a = \frac{C}{100} - 13$

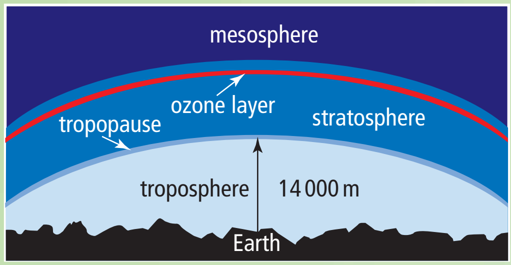
a) Tom is an active 14-year-old. What is the recommended number of Calories he should consume?

b) Juan is a moderately active 14-year-old boy. If he consumes 2831 Calories per day, is this greater or fewer Calories than the recommended amount?

c) The recommended requirement for a moderately active 14-year-old girl is 2100 Calories. Model this energy requirement by determining the value for x in the equation $a = \frac{C}{100} - x$.

MATH LINK

Meteorologists rely on models of our atmosphere to help them understand temperature and pressure differences, humidity, and a wide range of other variables. An important part of our atmosphere is the troposphere. It is the lowest layer of the atmosphere, where humans live and where weather occurs.

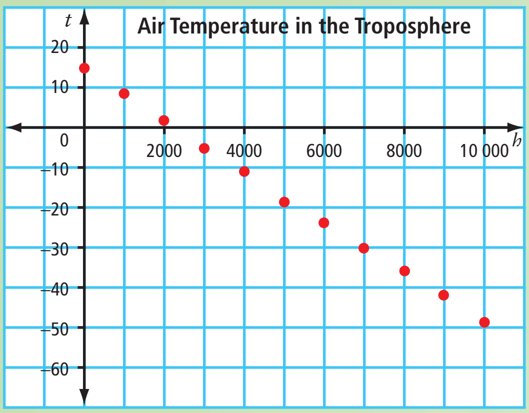


The equation that models air temperature change in the troposphere is $t = 15 - \frac{h}{154}$, where t is the temperature, in degrees Celsius, and h is the altitude, in metres.

a) What patterns do you see in the graph?

b) What connections do you see between the graph and the equation?

c) At what height in the troposphere is the temperature 0 °C?



10.4 Modelling and Solving Two-Step Equations: $a(x + b) = c$

Focus on...

After this lesson, you will be able to...

- ❑ model problems with two-step linear equations
- ❑ solve two-step linear equations and show how you worked out the answer

Kia plans to make a square Star Quilt for her grandmother. The quilt will have a 4-cm wide border around it. Kia wants the perimeter of the completed quilt to be 600 cm. How can Kia decide how long each side of the quilt should be before she adds the border?

Did You Know?

The centre of a Star Quilt is in the shape of the traditional eight-pointed morning star of the Lakota and Dakota Sioux. The Star Quilt is a symbol of tradition to the Plains peoples.

Explore the Math

Materials
- algebra tiles

How do you solve equations of the form $a(x + b) = c$?

Viktor missed yesterday's math class. Jackie will show him how to model and solve the equation $3(x - 5) = -6$

1. **a)** Use a variable tile to represent x.

b) How will you use negative 1-tiles to represent -5?

2. **a)** How many sets of $x - 5$ will you include in your model? Explain.

b) How will you complete your model of the equation?

3. **a)** What is the first thing you do to isolate the variable tile?

b) What equation does your model represent now?

c) What do you need to do to solve the equation?

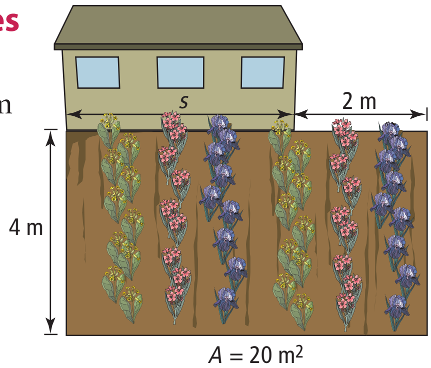
4. What is the unknown value of x?

Reflect on Your Findings

5. What steps did you take to solve the equation?

Example 1: Model With Algebra Tiles

A flower garden is in the shape of a rectangle. The length of the garden is 2 m longer than the length of the shed beside it. The width of the garden is 4 m. If the area of the garden is 20 m², what is the length of the shed?

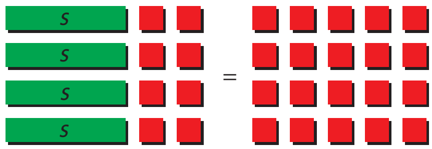


Solution

Let s represent the unknown length of the shed. The length of the garden can be represented by $s + 2$. The width of the garden is 4 m.
The equation that models the area of the garden is $4(s + 2) = 20$.

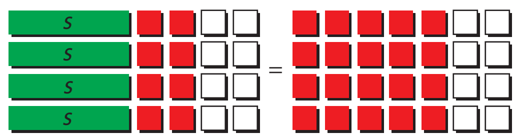
What is the formula for the area of a rectangle?

There are four groups of $(s + 2)$.
That means there are four variable tiles and eight positive 1-tiles on the left side of the equation.



To isolate the variable, subtract eight positive 1-tiles from both sides of the equal sign.

This is the same as adding eight negative 1-tiles to both sides of the equal sign.



There are now four variable tiles on the left side and 12 positive 1-tiles on the right side.

The four variable tiles must have the same value as the 12 positive 1-tiles.

Each variable tile must then have a value of three positive 1-tiles.

The length of the shed is 3 m.

Check:

$$\begin{aligned}\text{Left Side} &= 4(s + 2) \qquad \text{Right Side} = 20\\ &= 4(3 + 2)\\ &= 4(5)\\ &= 20\end{aligned}$$

Left Side = Right Side

The solution is correct.

Literacy Link

When substituting a value into an equation, be sure to use the correct order of operations:

- Brackets.
- Multiply and divide in order from left to right.
- Add and subtract in order from left to right.

Show You Know

Solve by modelling the equation.

a) $2(g + 4) = -8$ **b)** $3(r - 2) = 3$

Example 2: Solve Equations

Kia is making a square quilt with a 4-cm wide border around it. She wants the completed quilt to have a perimeter of 600 cm. What must the dimensions of Kia's quilt be before she adds the border?

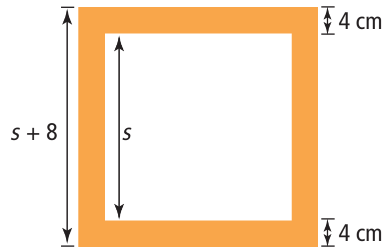
Strategies

Draw a Diagram

Solution

Let s represent the unknown side length of the quilt before the border is added. A border of 4 cm is added to each side. That means the side length of the quilt after the border is added is $s + 8$.

Model with the equation $4(s + 8) = 600$.



The length must be multiplied by 4 because there are four sides to the square quilt.

Method 1: Divide First

Isolate the variable s.

$$4(s + 8) = 600$$

$$\frac{4(s + 8)}{4} = \frac{600}{4}$$ Divide by 4 to undo the multiplication.

$$s + 8 = 150$$

$$s + 8 - 8 = 150 - 8$$ Subtract 8 to undo the addition.

$$s = 142$$

The quilt dimensions before adding the border should be 142 cm × 142 cm.

Literacy Link

The *distributive property* states that $a(b + c)$ equals $a \times b + a \times c$.

Method 2: Use the Distributive Property First

Isolate the variable s.

$$4(s + 8) = 600$$

$$4s + 32 = 600$$ Multiply both s and 8 by 4.

$$4s + 32 - 32 = 600 - 32$$ Subtract 32 from both sides of the equation.

$$4s = 568$$

$$\frac{4s}{4} = \frac{568}{4}$$ Divide both sides of the equation by 4.

$$s = 142$$

To use the distributive property, multiply the terms in the brackets by 4.

$$\begin{aligned} 4(s + 8) &= 4 \times (s + 8) \\ &= (4 \times s) + (4 \times 8) \\ &= 4s + 32 \end{aligned}$$

The quilt dimensions before adding the border should be 142 cm × 142 cm.

Check:

$$\begin{aligned} \text{Left Side} &= 4(s + 8) \qquad \text{Right Side} = 600 \\ &= 4(142 + 8) \\ &= 4(150) \\ &= 600 \end{aligned}$$

Left Side = Right Side

The solution is correct.

Show You Know

Solve each equation.

a) $-2(x - 3) = 12$ **b)** $-20 = 5(3 + p)$

Key Ideas

- To solve an equation, isolate the variable on one side of the equal sign.
- When undoing the operations performed on the variable, use opposite operations.
- Solve an equation of the form $a(x + b) = c$ by dividing first or by using the distributive property first.

Divide First:

$$
\begin{aligned}
-4(x - 7) &= 16 \\
\frac{-4(x - 7)}{-4} &= \frac{16}{-4} && \text{Divide by } -4 \text{ to undo the multiplication.} \\
x - 7 &= -4 \\
x - 7 + 7 &= -4 + 7 && \text{Add 7 to undo the subtraction.} \\
x &= 3
\end{aligned}
$$

Use the Distributive Property First:

$$
\begin{aligned}
-4(x - 7) &= 16 \\
-4x + 28 &= 16 && \text{Use the distributive property.} \\
-4x + 28 - 28 &= 16 - 28 && \text{Subtract 28 to undo the addition.} \\
-4x &= -12 \\
\frac{-4x}{-4} &= \frac{-12}{-4} && \text{Divide by } -4 \text{ to undo the multiplication.} \\
x &= 3
\end{aligned}
$$

- One method you can use to check your answer is substituting it back into the equation. Both sides of the equation should have the same value.

$$
\begin{aligned}
\text{Left Side} &= -4(x - 7) \qquad \text{Right Side} = 16 \\
&= -4(3 - 7) \\
&= -4(-4) \\
&= 16 \\
&\text{Left Side} = \text{Right Side}
\end{aligned}
$$

The solution is correct.

Communicate the Ideas

1. Draw diagrams to show how you would solve the equation $4 = 2(v - 3)$ using algebra tiles. Explain each step in words.

2. Julia and Chris are solving the equation $-18 = -6(x + 2)$. Is either strategy correct? Explain.

Julia: First, I subtract 2 from both sides. Then, I divide both sides by –6.

Chris: I start by dividing –6(x + 2) by –6. Then, I subtract 2 from both sides.

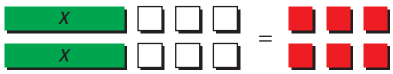
3. Describe a situation that can be modelled with the equation $2(r + 3) = -6$.

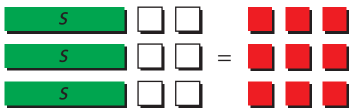
Check Your Understanding

Practise

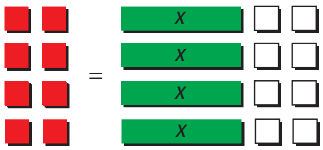
For help with #4 to #7, refer to Example 1 on page 395.

4. Solve the equation modelled by each diagram. Check your solution.

a) 

b) 

5. Solve the equation represented by each diagram. Verify your answer.

a) 

b) 

6. Model and then solve each equation. Check your answer.

a) $3(t - 2) = 12$ **b)** $6(j - 1) = -6$

7. Model and then solve each equation. Verify your solution.

a) $2(3 + p) = 8$ **b)** $0 = 7(n - 2)$

For help with #8 and #9, refer to Example 2 on page 396.

8. Solve each equation. Check your answer.

a) $6(r + 6) = -18$

b) $4(m - 3) = 12$

c) $3(1 + g) = -75$

d) $36 = 6(f + 13)$

9. Solve. Verify your solution.

a) $-21 = 3(k + 3)$

b) $42 = -14(n - 11)$

c) $8(x - 7) = -32$

d) $-10 = -5(w + 13)$

Apply

10. Show whether $x = -4$ is the solution to each equation.

a) $-8(x - 1) = 24$

b) $3(-8 - x) = -24$

c) $25 = -5(x - 1)$

d) $66 = 6(x + 7)$

11. The fence around Gisel's new tree is in the shape of an equilateral triangle. Gisel wants to increase the length of each side by 7 cm. The perimeter of her new fence will be 183 cm.

a) Let s represent the side length of the old fence. What equation models this situation?

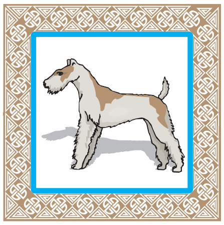
b) Determine the length of each side of the old fence.

12. The amount of food energy per day required by hikers is modelled by the equation $e = -125(t - 122)$, where e is the amount of food energy, in kilojoules (kJ), and t is the outside temperature, in degrees Celsius.

a) If the outside temperature is -20 °C, how much food energy is required per day?

b) If a hiker consumes 19 000 kJ of food energy based on the outside temperature, what is the temperature?

13. Barney wants to frame a square picture of his dog. The frame he bought fits a picture with a perimeter no greater than 96 cm. He plans to put a 2-cm blue border around the picture.



a) What equation models this situation?

b) Determine the maximum dimensions that the picture can have.

Extend

14. A computer rental company charges by the hour: \$5 for the first hour and \$4 for every hour after that. The fee rate can be modelled with the equation $4(n - 1) = T - 5$, where n is a number of hours greater than zero and T is the rental fee, in dollars. Candy's rental fee was \$17. For how many hours did she rent the computer?

15. A parking lot charges by the hour: \$2 for the first hour and \$3 for every hour after that. The formula used to calculate the number of hours someone has parked is $3(h - 1) = T - 2$, where h represents a number of hours greater than zero and T represents the total amount of the parking fee, in dollars. If Mark's parking fee is \$8, how long did he park in the lot?

16. The distance between Andrew's house and his grandfather's apartment is 42 km.

a) If Andrew rides his bike 2 km/h faster than his current speed, he could get there in 3 h. What is Andrew's current speed?

b) If Andrew wants to get there in 2 h, how much faster than his current speed should he ride his bike?

c) Do you think Andrew can get there in 2 h? Explain.

MATH LINK

Some jobs require working the night shift, such as from midnight to 8:30 a.m. Other jobs require working in isolated areas or under hazardous conditions. Depending on the job, the wage may be increased by a certain amount per hour or per month. This increase is called a premium.

a) Research and describe three different jobs that pay hourly or monthly wages plus a premium.

b) For each job, model the pay using an equation.

10 Chapter Review

Key Words

For #1 to #7, choose the word from the list that goes in each blank.

variable *distributive property* *equation*
linear equation *constant* *numerical coefficient*
opposite operations

1. A letter that represents an unknown number is called a(n) _____.

2. A mathematical statement with two expressions that have the same value is called a(n) _____.

3. Multiplication and division are _____ _____.

4. A number that multiplies the variable is called a(n) _____ _____.

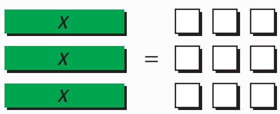
5. $5(b + 3) = 5 \times b + 5 \times 3$ is an example of how you use the _____ _____.

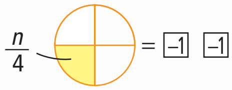
6. A number that does not change and that is added or subtracted from the value of an expression is called a(n) _____.

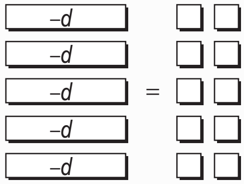
7. An equation that, when graphed, results in points that lie along a straight line is called a(n) _____ _____.

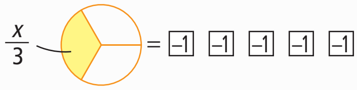
10.1 Modelling and Solving One-Step Equations: $ax = b$, $\frac{x}{a} = b$, pages 370–379

8. Solve the equation modelled by each diagram. Check your solution.

a)


b)


c)


d)


9. Solve by inspection.

a) $-22 = -11x$ **b)** $6r = -18$

c) $-8 = 2z$ **d)** $-5t = 15$

10. Solve each equation. Check your answer.

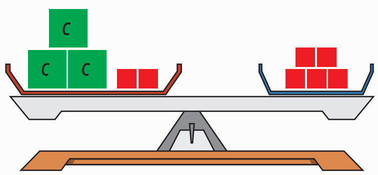
a) $-5 = \frac{p}{3}$ **b)** $\frac{n}{-11} = 3$

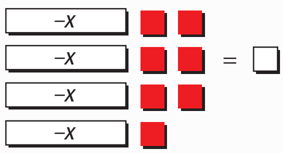
c) $-9 = \frac{x}{-4}$ **d)** $\frac{a}{-2} = -7$

11. Write two different equations that have a solution of 5 and that can be solved using multiplication or division.

10.2 Modelling and Solving Two-Step Equations: $ax + b = c$, pages 380–387

12. Write and solve the equation modelled by each diagram. Check your solution.

a)


b)


13. Show whether $x = -5$ is the solution to each equation.

a) $-7x - 2 = 33$ b) $4 - 3x = 19$

c) $-28 = 5x - 3$ d) $30 = 2x + 20$

14. Solve each equation. Check your solution.

a) $-3t + 8 = 20$ b) $5j - 2 = -127$

c) $-12 + 9p = 24$ d) $130 = 12n - 5$

15. Zoë has a collection of CDs and DVDs. The number of CDs she has is three fewer than four times the number of DVDs. Zoë has 25 CDs.

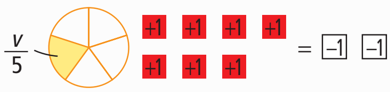
a) Choose a variable to represent the number of DVDs Zoë has. Write an equation that represents this situation.

b) How many DVDs does Zoë have?

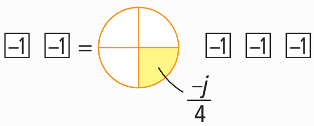
10.3 Modelling and Solving Two-Step Equations: $\frac{x}{a} + b = c$, pages 388–393

16. Solve the equation modelled by each diagram. Check your solution.

a)



b)



17. Identify the first operation and the second operation you should perform to solve each equation.

a) $\frac{t}{-3} + 13 = 9$ b) $\frac{r}{15} - 7 = -11$

c) $2 - \frac{x}{22} = 17$ d) $13 = -16 - \frac{h}{4}$

18. Solve. Verify your answer.

a) $3 - \frac{v}{-3} = 7$ b) $\frac{d}{3} - 13 = -8$

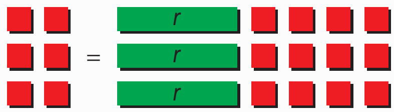
c) $17 = -4 + \frac{x}{-2}$ d) $-2 = \frac{n}{4} - 11$

19. According to the Canadian Soccer Association, in 2006, Saskatchewan's number of registered players was 1120 fewer than $\frac{1}{5}$ the number of soccer players registered in British Columbia. Saskatchewan had 23 761 registered soccer players that year. Write and solve an equation to determine how many players British Columbia had.

10.4 Modelling and Solving Two-Step Equations: $a(x + b) = c$, pages 394–399

20. Solve the equation modelled by each diagram. Check your solution.

a)



b)

21. Solve. Verify your solution.

a) $6(q - 13) = -24$ b) $-14 = 2(g + 4)$

c) $-18 = -6(k + 17)$ d) $16 = -4(x - 5)$

22. Diane wishes to create a square Star Quilt like the one shown. There will be a 3-cm border around the quilt and the perimeter of the completed quilt will be 372 cm. Write and solve an equation to determine the dimensions of the quilt before she adds the border.

23. Each side of a regular octagon is decreased by 3 cm. If the perimeter of the new octagon is 48 cm, what was the measure of each side of the original octagon?

10 Practice Test

For #1 to #5, choose the best answer.

1. What is the solution to $\frac{x}{3} = -12$?

A $x = 36$ **B** $x = 4$
C $x = -4$ **D** $x = -36$

2. The force, F, in newtons, required to stretch a spring a distance, d, in centimetres, is represented by the equation $F = 15d$. If a force of 38 N is used, how far will the spring stretch, to the nearest tenth of a centimetre?

A 0.3 cm **B** 0.4 cm
C 2.5 cm **D** 2.6 cm

3. What is the solution to $5n - 7 = -4$?

A $n = \frac{3}{5}$ **B** $n = \frac{4}{5}$
C $n = \frac{11}{5}$ **D** $n = \frac{31}{5}$

4. Which of these equations has the solution $p = -6$?

A $\frac{p}{3} - 4 = -2$ **B** $\frac{p}{3} + 4 = -2$
C $\frac{p}{-3} - 4 = -2$ **D** $\frac{p}{-3} + 4 = -2$

5. Wanda solved the equation $4(x - 3) = 2$ like this:

$4(x-3) = 2$
Step 1 $4x - 12 = 8$
Step 2 $4x = 20$
Step 3 $x = 5$

At which step did Wanda make her first mistake?

A Step 1 **B** Step 2
C Step 3 **D** No mistake was made.

Complete the statements in #6 and #7.

6. The opposite operation of division is ____.

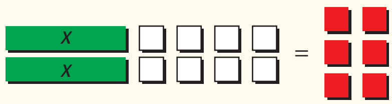
7. The solution to $-4(y + 10) = 24$ is $y =$ ____.

Short Answer

8. a) Draw a diagram that models the equation $-3x - 4 = 2$.

b) What is the solution to this equation?

9. Dillon used algebra tiles to model a problem.



a) What equation is being modelled?

b) What is the first step that Dillon should take to solve the equation using the algebra tiles?

10. Solve each equation. Verify your solution.

a) $4x = 48$ **b)** $\frac{t}{-5} = -8$

c) $2k - 6 = 31$ **d)** $\frac{d}{7} - 5 = 16$

e) $3 - \frac{n}{4} = 8$ **f)** $12 = 4(x - 2)$

11. a) Describe the steps you would take to solve the equation $-3(b + 3) = -15$.

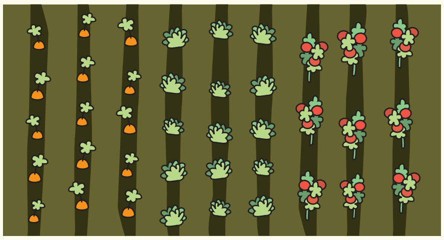
b) How are these steps different from the steps you would take to solve the equation $-3b + 3 = -15$?

12. The surface elevation of Lake Louise is 1536 m. This elevation is 45 m higher than seven times the elevation of Lake Athabasca.

a) Choose a variable to represent the elevation of Lake Athabasca. Write an equation to model this situation.

b) What is the elevation of Lake Athabasca?

13. The length of a rectangular vegetable garden is to be increased by 3 m. The new garden will have an area of 90 m². Write and then solve an equation to determine the length of the original garden.



5 m

Extended Response

14. a) What is wrong with the method used to solve the following equation?

$$\begin{aligned} -6 &= 18 + 3x \\ -6 + 18 &= 18 - 18 + 3x \\ 12 &= 3x \\ 4 &= x \end{aligned}$$

b) What is the correct method?

15. The formula for the perimeter of a rectangle is $P = 2(l + w)$, where P is the perimeter, l is the length, and w is the width of the rectangle. The perimeter of the rectangle shown is 14 cm.

$w = 3$ cm

a) What is the length of the rectangle? Check your solution.

b) Another rectangle has the same length as the rectangle shown but a perimeter of 12 cm. What is the area of this rectangle?

Wrap It Up!

Report on how different linear equations could be used in everyday situations. Include all five of these types of linear equations:

$ax = b$ $\quad \frac{x}{a} = b, a \neq 0$ $\quad ax + b = c$

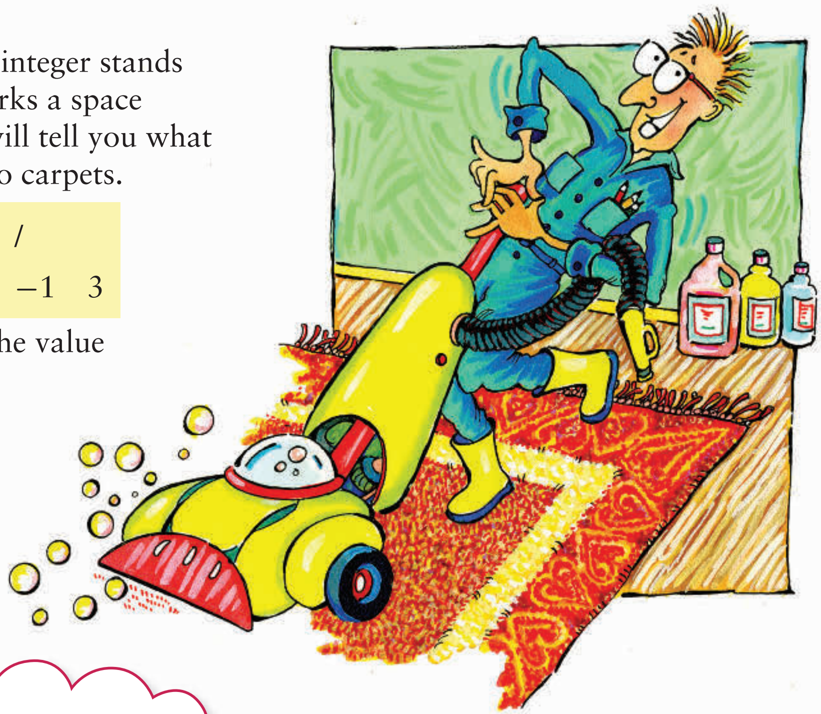
$\frac{x}{a} + b = c, a \neq 0$ $\quad a(x + b) = c$

In your report,

- describe a different situation or job for each of the five linear equations
- identify what each variable, constant, and numerical coefficient represents in each of your equations
- solve each of your linear equations, using values appropriate for the situation or job
- identify how one of your equations may change based on the circumstances

Math Games

Rascally Riddles



1. In the following message, each integer stands for a letter. The "/" symbol marks a space between words. The message will tell you what mathematicians use to shampoo carpets.

−4	2	4	4	6	−4	−3	/	
3	2	−6	−7	−3	1	2	−1	3

a) Solve the equations to find the value of each variable.

$2c = -8$

$\frac{e}{3} = 2$

$4i - 3 = 1$

$\frac{l}{3} + 4 = 2$

$2(n + 1) = 0$

$5o - 1 = 9$

The variable in this equation is the letter o. This letter is not commonly used as a variable because it looks like the integer 0.

$\frac{r}{-4} + 3 = 2$

$3(s - 5) = -6$

$4t + 7 = -5$

$\frac{u}{-7} = 1$

b) Replace each integer in the message by the variable with this value. What do mathematicians use to shampoo carpets?

2. a) As a class or in a group, brainstorm how you would write a riddle like the one in #1.

b) Write a riddle of your own. It must include
- a short message made with integers that stand for letters
- a set of equations that can be solved to determine the letters that will replace the integers

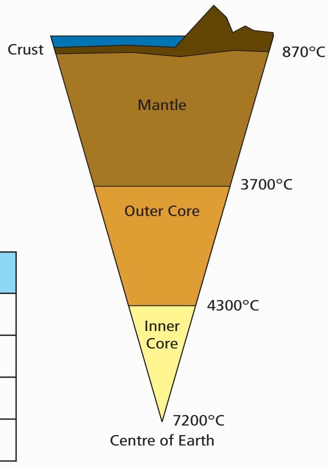
c) Check that your equations give your intended message.

d) Have a classmate solve your riddle.

Challenge in Real Life

The Earth's Core

Earth is made up of several distinct layers. The deeper layers are hotter and denser because the temperature and pressure inside Earth increases with depth. The table below provides information concerning how the temperature increases with depth.



Materials

- grid paper

Layer	Depth (km)	Temperature (°C)
Crust	–90	870
Mantle	–2921	3700
Outer core	–5180	4300
Inner core	–6401	7200

1. Graph the data from the table. Label your graph. Note: Make sure that your y-axis goes to at least 9000 °C, since you will need this value to complete #3b).

2. a) Calculate the total temperature change moving from the lower part of the crust to the centre of Earth.
   b) What is the total depth change moving from the lower part of the crust to the centre of Earth? Show your work.
   c) What is the approximate change in temperature for every kilometre you go into Earth? Justify your answer.

3. The approximate temperature by depth can be modelled using the linear equation $T = d + 870$, where d is the depth, in kilometres, and T is the temperature, in degrees Celsius.
   a) Use the equation to calculate the approximate temperature at a depth of 3400 km. Show your calculations. Verify your solution by placing an x on your graph from #1.
   b) Use the equation to identify the depth at which the temperature is approximately 9000 °C. Show your calculations and verify your solution by placing a y on your graph from #1.
   c) How does your answer to #3b) compare to your answer in #2c)? Explain why there may be some differences.