Inductive Reasoning

Lesson #1

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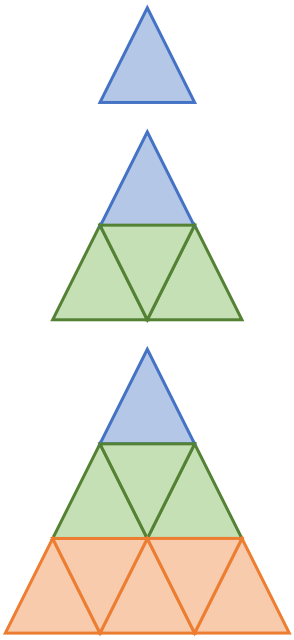
Problem

Brooklyn, a fabric artist, is going to be sewing a quilt using equilateral triangles. Brooklyn needs to know how many triangles to cut if the final pattern is 10 rows tall. Brooklyn comes up with the following relationship.

1st Row	2nd Row	3rd Row	10th Row
1 Triangle	4 Triangles	9 Triangles	? Triangles



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Conjecture

- Is a testable expression that is based on available information (i.e. evidence), but is not yet proved.
- ”Mathematical” Hypothesis

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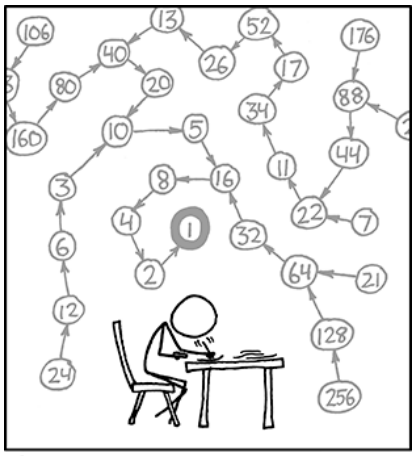
Famous Math Conjectures



THE COLLATZ CONJECTURE STATES THAT IF YOU PICK A NUMBER, AND IF IT'S EVEN DIVIDE IT BY TWO AND IF IT'S ODD MULTIPLY IT BY THREE AND ADD ONE, AND YOU REPEAT THIS PROCEDURE LONG ENOUGH, EVENTUALLY YOUR FRIENDS WILL STOP CALLING TO SEE IF YOU WANT TO HANG OUT.

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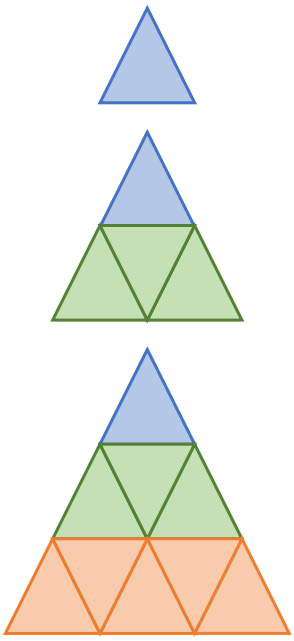
- Is there a pattern in the table? Do we need more information?
- What is your conjecture?
- Can we test your conjecture?



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Example

Write a conjecture that describes the following sequence of numbers: 2, 4, 12, 48, 240. Then use your conjecture to find the next number in the sequence.

Step 1: What is the pattern?

2 4 12 48 240

x2 x3 x4 x5

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Example

2 4 12 48 240 ???

x2 x3 x4 x5 x6?

Step 2: Make a conjecture

The numbers are multiplied by 2, 3, 4, then 5. The next number in the sequence will be the product of 240 x 6 or 1440.

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Inductive Reasoning

- When we use patterns, trends, and properties to make a general prediction.

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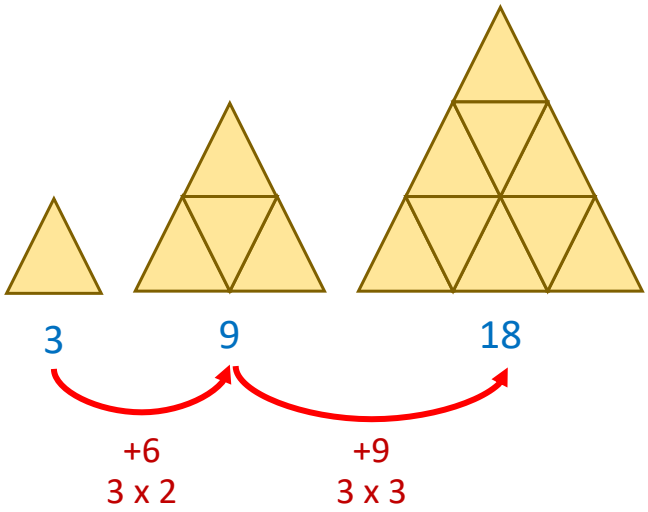
Example

Write a conjecture that describes the pattern shown. Then use your conjecture to find the next pattern in the sequence.

Step 1: What is the pattern?

Step 2 : Write a conjecture.

The figure will increase by the next multiple of 3. If we add 12, the figure is made of 30 segments



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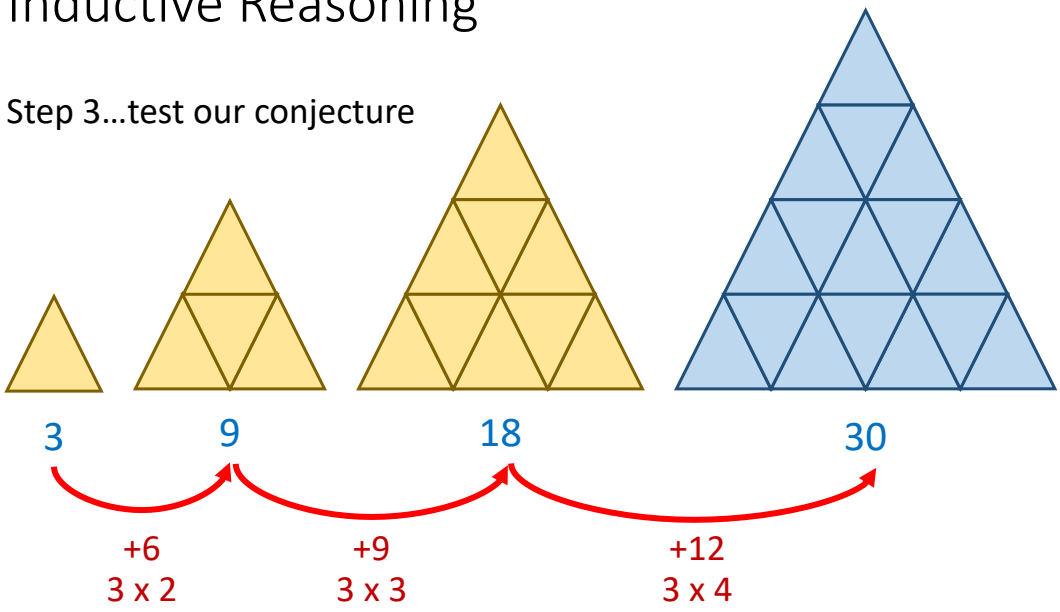
Inductive Reasoning

Step 3…test our conjecture



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Example

Write a conjecture that describes the pattern in the sequence 1, 1/4, 1/9, 1/16 and 1/25. Then use your conjecture to find the next term in the sequence.

Step 1: Look for pattern $\frac{1}{1^2}\ \frac{1}{2^2}\ \frac{1}{3^2}\ \frac{1}{4^2}\ \frac{1}{5^2}\ \frac{\mathbf{1}}{\mathbf{6^2}}$ or $\frac{\mathbf{1}}{\mathbf{36}}$

Step 2: Write a conjecture

The next term has a denominator that is the next "perfect square" or next integer squared.

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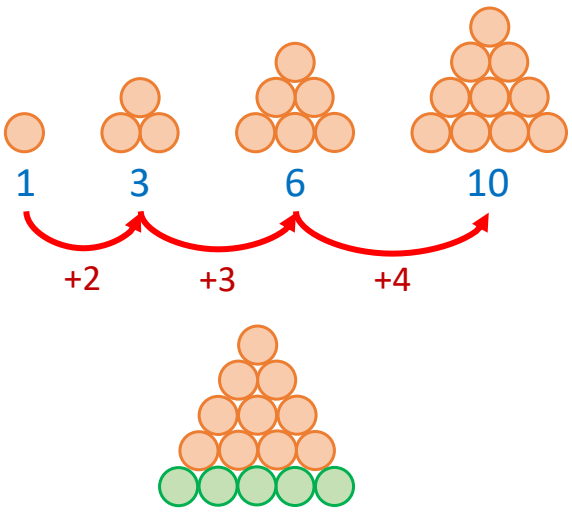
Example

Write a conjecture that describes the pattern shown. Then use your conjecture to find the next pattern in the sequence.



Step 1: What is the pattern?

Step 2 : Write a conjecture.

The next figure in the sequence will increase by 5 more, or 10 + 5 = 15

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Example

Write a conjecture about the sum of an odd number and an even number. List some examples in your conjecture.

3 + 4 = 7; 1 + 8 = 9; 21 + 32 = 54; 121 + 44 = 165; 3491 + 568 = 4059

Conjecture:

The answer will always be an odd number.

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Validity of Conjectures

Some conjectures may initially seem valid,, but are later shown not to be valid after more evidence is gathered.

As a result…The best we can say about a conjecture reached through Inductive Reasoning is that there is evidence either to support or deny it.

- The conjecture may need to revised and new evidence is gathered.

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Problem

Taylor found an interesting numeric pattern. Taylor makes a conjecture that the following pattern will continue on forever.

Is Taylor's conjecture correct?

$$1 \times 8 + 1 = 9$$
$$12 \times 8 + 2 = 98$$
$$123 \times 8 + 3 = 987$$
$$1234 \times 8 + 4 = 9876$$

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Is Taylor's conjecture correct?

$$1 \times 8 + 1 = 9$$
$$12 \times 8 + 2 = 98$$
$$123 \times 8 + 3 = 987$$
$$1234 \times 8 + 4 = 9876$$

$$123456789 \times 8 + 9 = 987654321$$

$$12345678910 \times 8 + 10 = 9876543\mathbf{1}\mathbf{2}\mathbf{9}\mathbf{0}$$
$$1234567890 \times 8 + 10 = 9876543\mathbf{1}\mathbf{3}\mathbf{0}$$
$$12345678910 \times 8 + 0 = 9876543\mathbf{1}\mathbf{2}\mathbf{8}\mathbf{0}$$
$$1234567890 \times 8 + 0 = 9876543\mathbf{1}\mathbf{2}\mathbf{0}$$

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Counterexamples to a Conjecture

- A Counterexample is an example that invalidates or makes a conjecture false.

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Examples

Find a counterexample to that makes the following conjecture false:

- If n is a real number, then $n^2 > n$

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Questions

Section 1.1, Page 12-14

#'s 3, 7, 9, 11 & Math in Action (page 15)

Section 1.3, Page 22-25

#'s 3, 5, 8, 10, 19, 21 & Reasoning in Science (page 24)

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