Deductive Reasoning

Lesson #2

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Problem

Pick any positive whole number and add 4 to it. Find the sum of the new number and add the original number. Next add 6 to the sum. Divide the new sum by 2. Lastly, subtract the original number from the quotient. **What number do you get?**

10 + 4 = 14
14 + 10 = 24
24 + 6 = 30
30 ÷ 2 = 15
15 – 10 = **5**

Conjecture

Picking any positive whole number will result in a value of 5.

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Deductive Reasoning

When we make specific conclusion through logical reasoning using general assumptions that are known to be valid.

May be false, but is logically valid.

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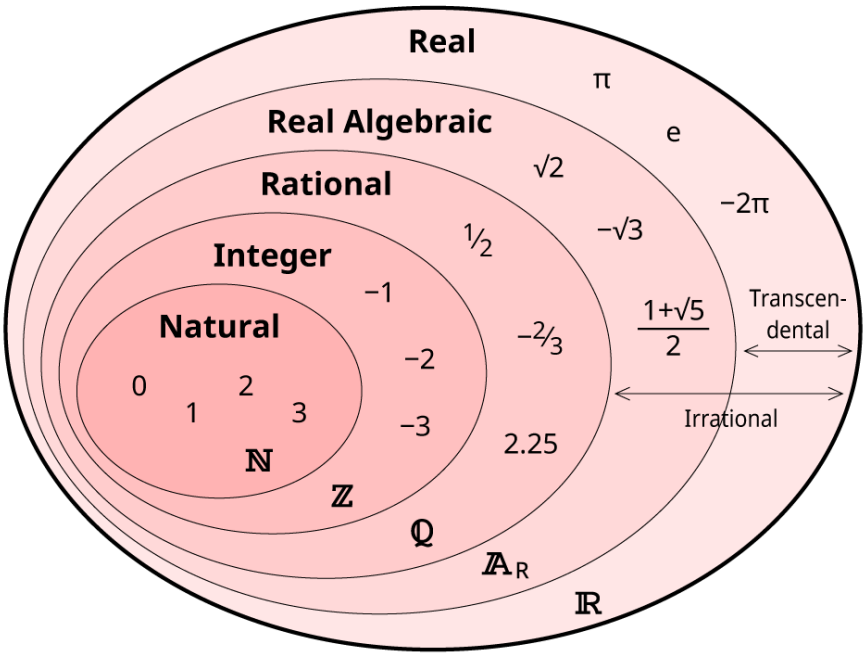

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Problem Revised

Pick any positive whole number and add 4 to it. Find the sum of the new number and add the original number. Next add 6 to the sum. Divide the new sum by 2. Lastly, subtract the original number from the quotient. **What number do you get?**

$10 + 4 = 14$

$14 + 10 = 24$

$24 + 6 = 30$

$30 \div 2 = 15$

$15 - 10 = \boxed{5}$

Conjecture

Picking any Real number will result in a value of 5.

- What happen if you pick an integer (i.e. a negative whole number)?
- What if you pick a rational number (i.e. fractions or decimals)?
- What if you pick an irrational number (i.e. π, 1/3, etc…)

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Generalizations & Proofs

Generalization

A principle, statement, or idea that has general application

Proof

A mathematical argument shown that a statement is valid in all cases, or that no counterexamples exists.

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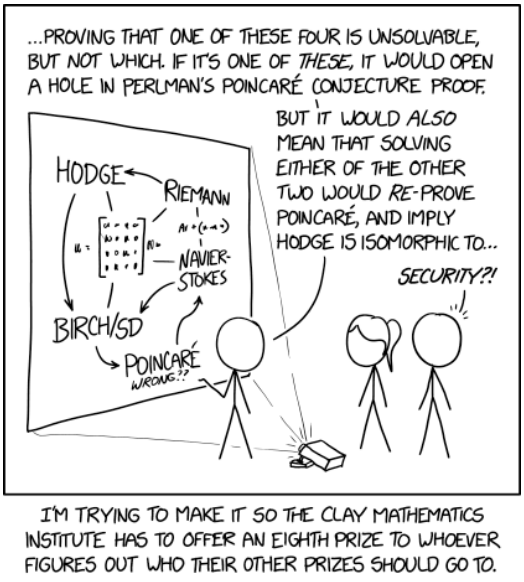
Famous Math Conjectures

- Fermat's Last Theorem
- Twin Prime Conjecture
- Four color theorem
- Collatz Conjecture
- Beal Conjecture
- Riemann Hypothesis
- Poincaré Conjecture (2003)
- P versus NP Conjecture
- Hodge Conjecture
- Yang–Mills existence and mass gap
- Navier–Stokes existence and smoothness
- Birch and Swinnerton-Dyer Conjecture



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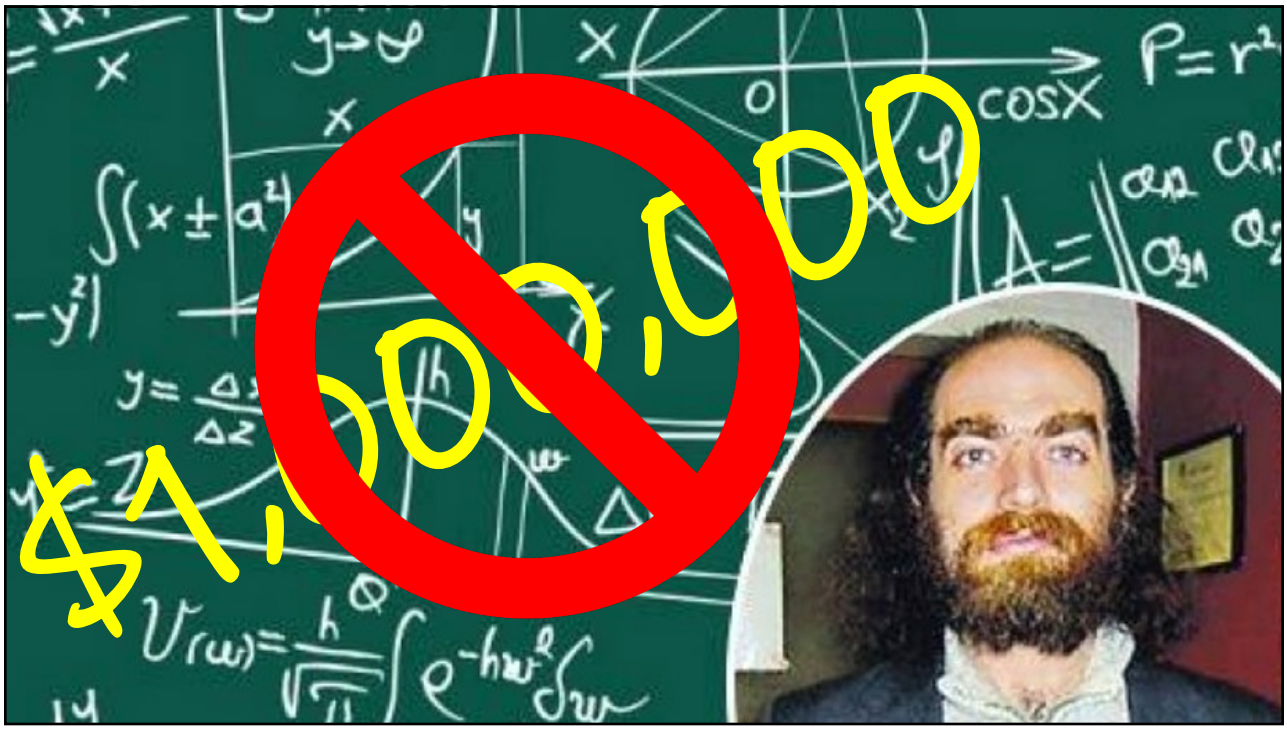

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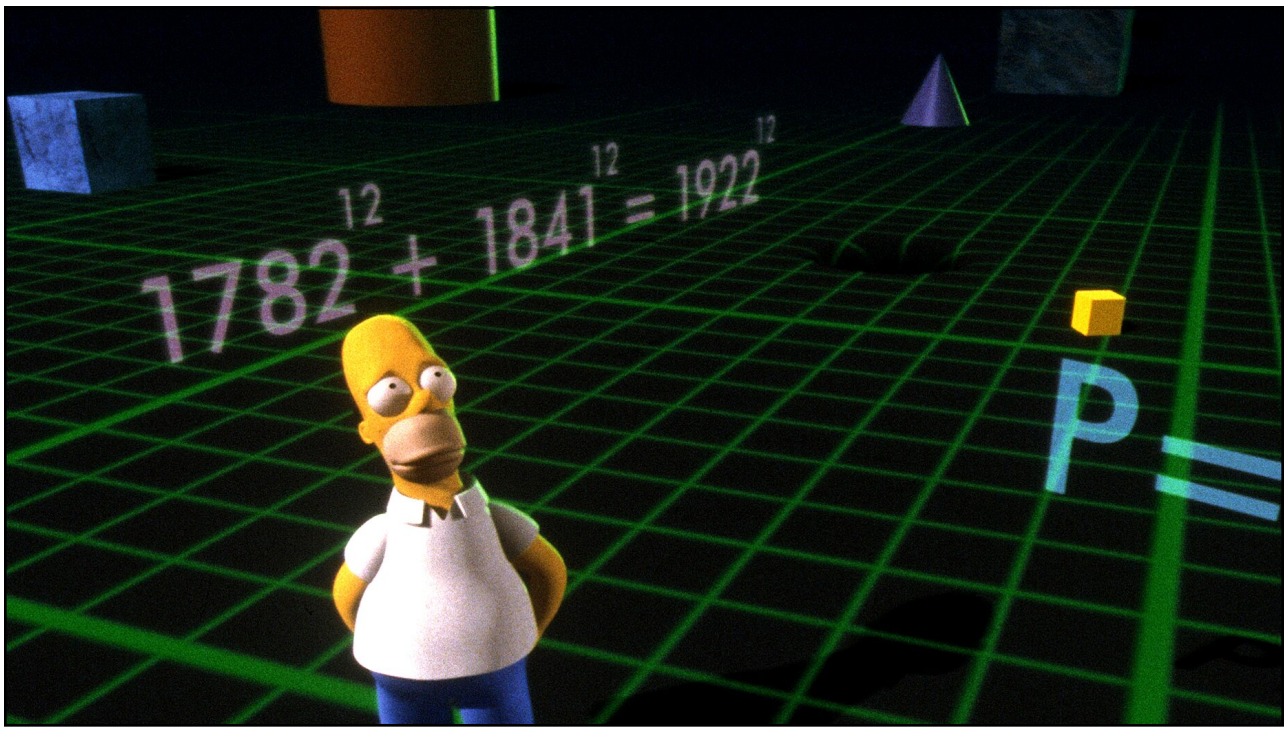
$1782^{12} + 1841^{12} = 1922^{12}$
P =

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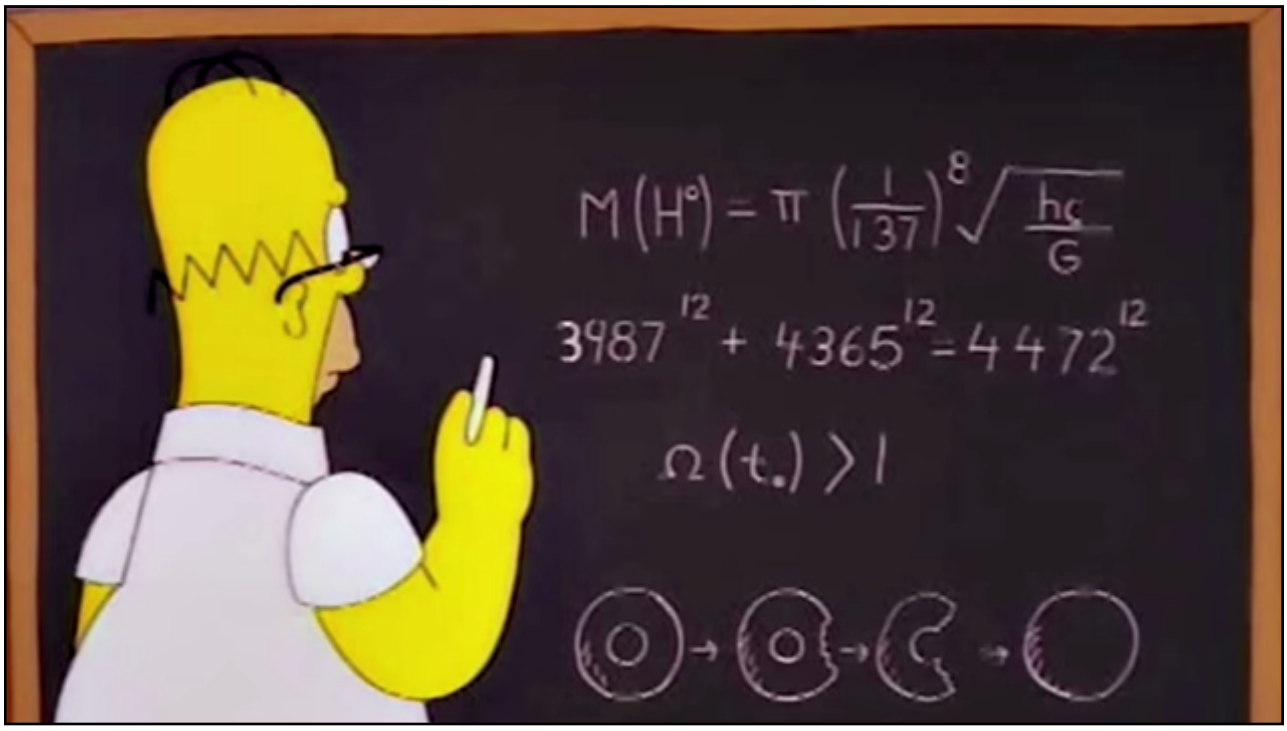
$M(H^\circ) = \pi \left(\frac{1}{137}\right)^8 \sqrt{\frac{hc}{G}}$
$3987^{12} + 4365^{12} = 4472^{12}$
$\Omega(t_0) > 1$

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Problem Revisited

Pick any positive whole number and add 4 to it. Find the sum of the new number and add the original number. Next add 6 to the sum. Divide the new sum by 2. Lastly, subtract the original number from the quotient. **What number do you get?**

Proof

Picking any number 'n' will always result in the value of 5.

$$n + 4 = n + 4$$
$$(n + 4) + n = 2n + 4$$
$$(2n + 4) + 6 = 2n + 10$$
$$(2n + 10) \div 2 = n + 5$$
$$n + 5 - n = \boxed{5}$$

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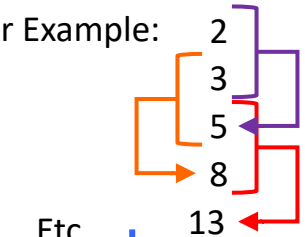
Test of Genius

Pick an two single digit numbers... 0 – 9 (except for double zero's) . Next, add them together to create a new number. Next add the new number to the previous number to create another new number...repeat until you have 10 numbers.

Next race to add up the 10 numbers.

For Example:



Etc...

2
3
5
8
13
21
34
55
89
144
233
605

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Test of Genius

Solution:

Shortcut...Take the 7th number in the sequence (even faster to select the 4th from the bottom) and multiple this number by 11!

Trick...for multiplying by 11

5 5

5 (5 + 5) 5

5 (1 0) 5

(5 + 1) 0 5 = 6 0 5

For Example:

2
3
5
8
13
21
34
11 x 55
89
144
233
605

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Prove

Instruction	*Test Case 1*	*Test Case 2*	*Test Case 3*
Choose a number less than 10	7	3	9
Add 7	14	10	16
Multiply by 2	28	20	32
Subtract the original number	21	17	23
Subtract 2	19	15	21
Subtract the original number	12	12	12

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Prove

The sum of 3 consecutive integers is a multiple of 3?

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Activity

Create your own ‘magical’ system to create any number of your choice regardless of the starting number chosen.

System must consist of at least 5 steps and include an assortment of mathematical operations.

Provide 3 samples of evidence to validate your system and a deductive proof.

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Inductive vs. Deductive

Deductive

- Certainty
- Guaranteed Conclusion
- Proof

Reasoning

- Categorical Reasoning
- Truth-Functional Reasoning

Inductive

- Reasonable Confidence
- Likely Conclusion
- Probability

Reasoning

- Analogical Reasoning
- Explanatory Reasoning

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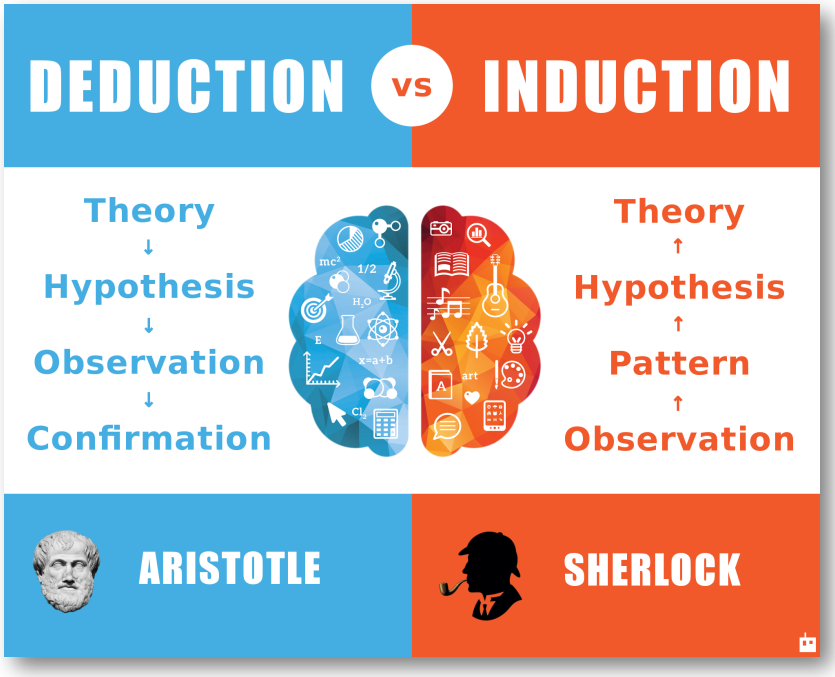

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Proofs That Are Not Valid

An Invalid Proof is a proof that contains an error in reasoning or that contains an invalid assumption.

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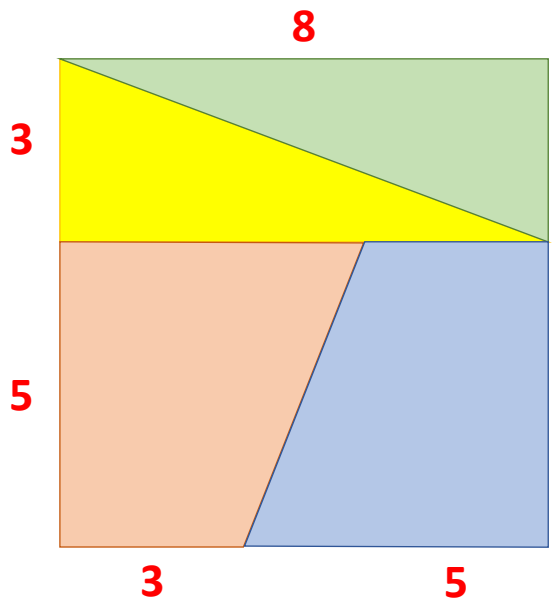
Example

Draw and Cut out the following shape as carefully as possible. Use Graph Paper!

- What is the area of this shape?



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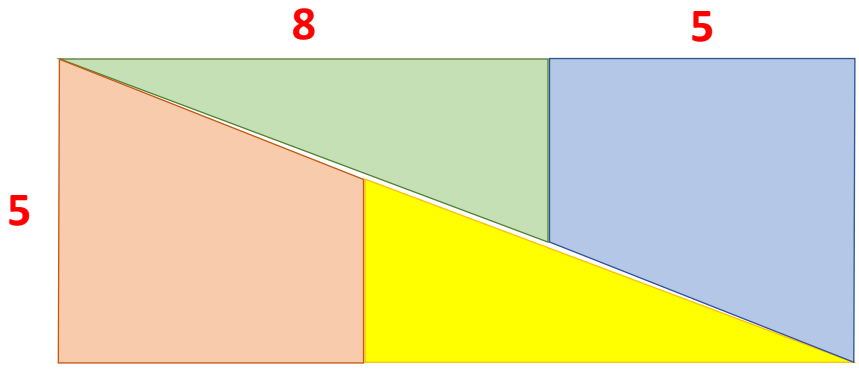
Example

Rearrange the shapes to create a different rectangle.

- What is the area of this new rectangle?



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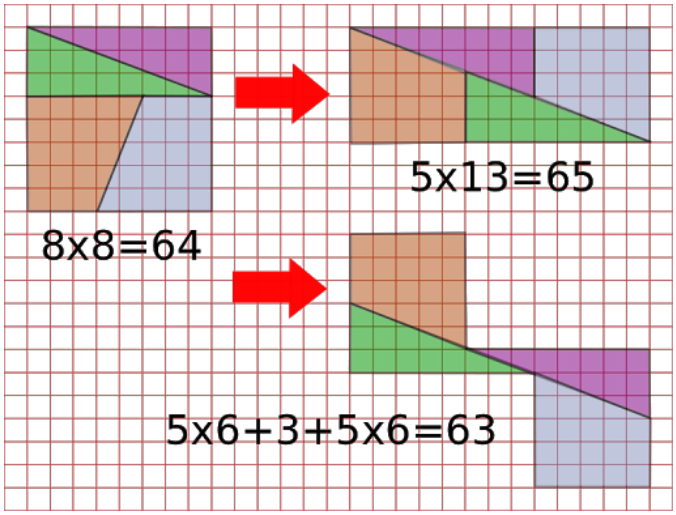
Example

- Did we just prove that.

  63 = 64 = 65????
- Does this contradict common sense?
- Where was the error? Prize to the first person who figures it out!



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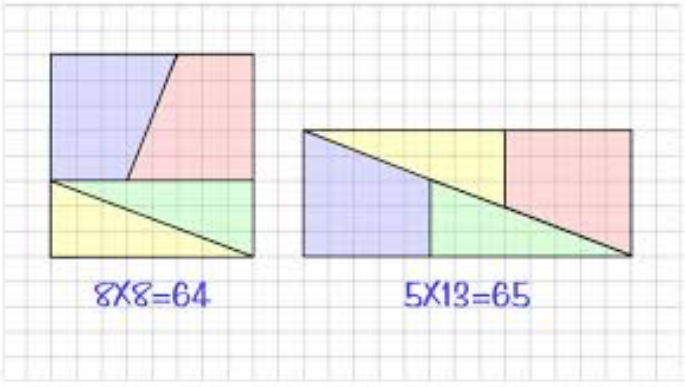
Example

- What is the real area?
- Is there anything special about the dimensions of the square (3, 5 & 13)?



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Example

Rebecca says she can prove that 2 = 0. Here is her proof. Let 'a' and 'b' be equal to 1.

Did she make an error in her reasoning? If yes, where?

$$a = b$$
$$a^2 = b^2$$
$$a^2 - b^2 = 0$$
$$(a - b)(a + b) = 0$$
$$\frac{(a - b)(a + b)}{(a - b)} = \frac{0}{(a - b)}$$
$$1(a + b) = 0$$
$$a + b = 0$$
$$1 + 1 = 0$$
$$2 = 0$$

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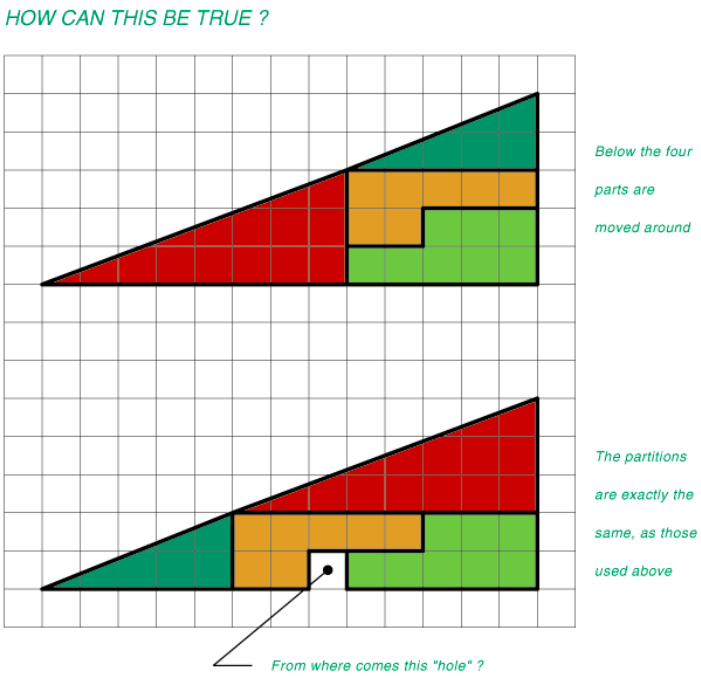
Example

- What is going here?



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The Scarecrow Conjecture

"The sum of the square root of any two sides of an isosceles triangle is equal to the square root of the remaining side."

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Questions

Section 1.4, Page 32-33

#'s 7, 9 & 19

Section 1.5, Page 43-44

#'s 5, 6, 7 & 10

Section 1.6, Page 49

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Also:

Create your own 'magical' system to create any number of your choice regardless of the starting number chosen.

System must consist of at least 5 steps and include an assortment of mathematical operations.

Provide 3 samples of evidence to validate your system and a deductive proof.

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