BONUS: Algorithm Trick! (Only works for Perfect Squares)
Here is a video of the trick: How to Calculate Square Roots

Step 1: Start by memorizing the squares of numbers from 0 to 9.
Step 2: Look at the last digit of the number whose square root you want to find.
Step 3: Match the last digit to the squares you've memorized. If the last digit matches a square's last digit, that square is your starting point.
Step 4: Cross out the last two digits of the number you're finding the square root for.
Step 5: Look for the square closest to the remaining number without exceeding it.
Step 6: If there are multiple possibilities, use the remaining digits to determine the correct square. You may have to test the squares in between to find the correct one.
Step 7: Once you've found the correct square, write down the corresponding digit.

a) $\sqrt{729}$ = 27^2 b) $\sqrt{4096}$ = 64^2 c) $\sqrt{7921}$ = 89^2

This trick also works similarly for perfect cubes.

a)
$3^2 = 9$
$7^2 = 49$
7~~29~~
2^2
23^2 or 27^2
$\boxed{27^2}$
pick a # in between
25^2 = trick
$2 \times 3 = 6 \rightarrow 625 < 729$
Need larger value

b)
$4^2 = 16$
$6^2 = 36$
40~~96~~
6^2
64^2 or 66^2
$\boxed{64^2}$
65^2
$6 \times 7 = 4225$
$4225 > 4096$
∴ must be smaller value!

c)
$1^2 = 1$
$9^2 = 81$
79~~21~~
8^2
81^2 or 89^2
$\boxed{89^2}$
85^2
$8 \times 9 = 7225$
$7225 < 7921$
∴ Need larger value!

Simplifying Roots

In math we typically don't like decimal approximations and prefer an exact value. To do this we find the prime factors of value inside the root and use the properties of square roots to simplify our answer.

Prime Factors

40
4 10
2 2 2 5

Let's consider the following example: $\sqrt{40}$

All Factors

What are the prime factors of 40? 1, 2, 4, 5, 8, 10, 20, 40

Replace 40 with the product of all the prime factors: $\sqrt{(2 \times 2 \times 2 \times 5)}$

Use the "**Jail Break**" Method: for every *PAIR* of identical factors one factor escapes in front of the square root sign and the other dies escaping ☹

$$\text{So } \sqrt{40} = \sqrt{(2 \times 2 \times 2 \times 5)} = 2\sqrt{(2 \times 5)} = 2\sqrt{10}$$

Examples:

a) $\sqrt{24}$ = ________

3 8
2 4
2 2

$\sqrt{2 \times 2 \times 2 \times 3}$

b) $\sqrt{54}$ = ________

6 9
2 3 3 3

$\sqrt{2 \times 3 \times 3 \times 3}$

c) $\sqrt{250}$ = ________

10 25
2 5 5 5

$\sqrt{2 \times 5 \times 5 \times 5}$

Cubed Roots

Similarly, we can apply the same method for cubed roots; however, this time we need *THREE* identical factors in order to pull on out.

Examples:

a) $\sqrt[3]{24} = 2\sqrt[3]{3}$

$\sqrt{2 \times 2 \times 2 \times 3}$

b) $\sqrt[3]{54} = 3\sqrt[3]{2}$

$\sqrt{2 \times 3 \times 3 \times 3}$

c) $\sqrt[3]{250} = 5\sqrt[3]{2}$

$\sqrt{2 \times 5 \times 5 \times 5}$

Practice:

Simplify the following.

a) $\sqrt{98} = 7\sqrt{2}$

98 → 2, 49; 49 → 7, 7

$\sqrt{2\times7\times7}$

b) $\sqrt{30} =$ Nope can't simplify

30 → 3, 10; 10 → 2, 5

$\sqrt{2\times3\times5}$

c) $\sqrt[3]{88} = 2\sqrt[3]{11}$

88 → 2, 44; 44 → 4, 11; 4 → 2, 2

$\sqrt[3]{2\times2\times2\times11}$

d) $\sqrt{63} = 3\sqrt{7}$

63 → 9, 7; 9 → 3, 3

$\sqrt{3\times3\times7}$

e) $\sqrt[3]{16} = 2\sqrt[3]{2}$

16 → 4, 4; 4 → 2, 2; 4 → 2, 2

$\sqrt[3]{2\times2\times2\times2}$

f) $\sqrt[3]{40} = 2\sqrt[3]{5}$

40 → 4, 10; 4 → 2, 2; 10 → 2, 5

$\sqrt[3]{2\times2\times2\times5}$

g) $\sqrt[3]{48} = 2\sqrt[3]{6}$

48 → 4, 12; 4 → 2, 2; 12 → 3, 4; 4 → 2, 2

$\sqrt[3]{2\times2\times2\times2\times3}$

h) $\sqrt{20} = 2\sqrt{5}$

20 → 4, 5; 4 → 2, 2

$\sqrt{2\times2\times5}$

i) $\sqrt[3]{32} = 2\sqrt[3]{4}$

32 → 4, 8; 4 → 2, 2; 8 → 2, 4; 4 → 2, 2

$\sqrt[3]{2\times2\times2\times2\times2}$

j) $\sqrt{48} = 4\sqrt{3}$

48 → 2, 24; 24 → 3, 8; 8 → 2, 4; 4 → 2, 2

$\sqrt{2\times2\times2\times2\times3}$

k) $\sqrt[3]{128} = 4\sqrt[3]{2}$

128 → 2, 64; 64 → 8, 8; 8 → 2, 2, 2; 8 → 2, 2, 2

$\sqrt[3]{2\times2\times2\times2\times2\times2\times2}$

l) $\sqrt[3]{144} = 2\sqrt[3]{18}$

144 → 12, 12; 12 → 3, 4; 12 → 4, 3; 4 → 2, 2; 4 → 2, 2

$\sqrt[3]{2\times2\times2\times2\times3\times3}$