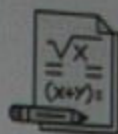


Integers - Squares, Cubes & Roots

Math 8



Method 2: Benchmarking a Root (Number Line)

In this approach, you identify the known lower and upper perfect squares and the difference between them and the root you would like to determine.

Step 1: Identify Lower and Upper perfect square. The lower value will tell you what whole number your value will start from.

Step 2: Determine the difference between the upper and lower values. This will be your denominator value.

Step 3: Determine the difference between the root you are trying to determine with either the lower OR upper limit. This will be your numerator.

Step 4: Divide your result from step 3 by your result in step 2 and add this value to your starting value in Step 1.

Practice: Estimate the square roots for the following using the benchmark method.

a) $\sqrt{19}$

Number line: $\sqrt{16}$ at 4, $\sqrt{25}$ at 5. $\sqrt{19}$ is between 4 and 5.

$\Delta = 25 - 16 = 9 \Rightarrow \frac{3}{9} = .\bar{3}$

$\Delta = 19 - 16 = 3$ or $25 - 19 = 6$

Result: $\sqrt{19} \approx 4.\bar{3}$

b) $\sqrt{31}$

Number line: $\sqrt{25}$ at 5, $\sqrt{36}$ at 6. $\sqrt{31}$ is between 5 and 6.

$\Delta = 36 - 25 = 11$

$\Delta = 31 - 25 = 6$

$\frac{6}{11} = 0.5454$

Result: $\sqrt{31} \approx 5.54$

c) $\sqrt{47}$

Number line: $\sqrt{36}$ at 6, $\sqrt{49}$ at 7. $\sqrt{47}$ is between 6 and 7.

$\Delta = 49 - 36 = 13$

$\Delta = 47 - 36 = 11$

$\frac{11}{13} = 0.84615\dots$

Result: $\sqrt{47} \approx 6.846$

Method 3: Using your Calculator!

Your calculator is equipped with a magically button square root button, that can do all of this for you!

Practice:

Use your calculator to determine the answer to the following to a maximum of 4 decimal places if required.

a) $\sqrt{81} = 9$

b) $\sqrt{-16} = \text{Error!}$

c) $-\sqrt{9} = -3$

d) $\sqrt{1} = 1$

e) $\sqrt{-16} = \text{Error!}$

f) $-\sqrt{9} = -3$

g) $-\sqrt{45} = -6.7082\dots$

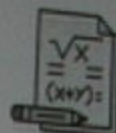
h) $\sqrt{2} = 1.4142\dots$

i) $-\sqrt{136} = -11.6619\dots$

j) $\sqrt{576} = 24$

k) $\sqrt{2116} = 46$

l) $\sqrt{46225} = 215$



Cubed Roots

The process is much the same for cubed root but granted more difficult. So, for cubed roots we only use our calculator!

Your calculator will be equipped with a button that looks like: $\sqrt[3]{\quad}$, $\sqrt[\times]{\quad}$, $\sqrt[\square]{\quad}$

Unlike square roots, a cubed root could negative (-) inside of the root sign.

Let us consider the following: $\sqrt[3]{(-27)}$

This means that the product of **three** identical numbers produces a negative (-) value. We know from our integer rules the only way to make a negative (-) is by multiplying an odd number of negative (-) values...

$$(+3) \times (+3) \times (+3) = +27 \rightarrow (+3)^3 = +27$$

$$(-3) \times (-3) \times (-3) = -27 \rightarrow (-3)^3 = -27 \text{ Bingo!}$$

$$(-3) \times (+3) \times (+3) = -27 \rightarrow \text{Bases are not the same, therefore no repeated multiplication, hence cannot be expressed as an exponent}$$

However, you can have a negative outside (i.e. in front) of the square root. You can think of this as the square root being multiplied by negative one (-1)

Example:

$$a) \sqrt[3]{(-8)} = \underline{-2}$$

$$b) \sqrt[3]{(-64)} = \underline{+4}$$

$$c) \sqrt[3]{(-125)} = \underline{-5}$$

Practice:

Use your calculator to determine the answer to the following to a maximum of 4 decimal places if required.

$$a) \sqrt[3]{120} = \underline{4.9524} \dots \quad b) \sqrt[3]{(-9)} = \underline{-2.0801} \dots \quad c) \sqrt[3]{27} = \underline{-3}$$

$$d) \sqrt[3]{1} = \underline{1} \quad e) \sqrt[3]{(-216)} = \underline{-6} \quad f) \sqrt[3]{(-64)} = \underline{+4}$$

$$g) \sqrt[3]{45} = \underline{3.5569} \quad h) \sqrt[3]{20} = \underline{2.7144} \dots \quad i) \sqrt[3]{343} = \underline{-7}$$