Method 2: Benchmarking a Root (Number Line)

In this approach, you identify the known lower and upper perfect squares and the difference between them and the root you would like to determine.

Step 1: Identify Lower and Upper perfect square. The lower value will tell you what whole number your value will start from.
Step 2: Determine the difference between the upper and lower values. This will be your denominator value.
Step 3: Determine the difference between the root you are trying to determine with either the lower OR upper limit. This will be your numerator.
Step 4: Divide your result from step 3 by your result in step 2 and add this value to your starting value in Step 1.

Practice: Estimate the square roots for the following using the benchmark method.

a) $\sqrt{19} = 4.\overline{3}$

$\sqrt{16}$ — $\sqrt{19}$ — $\sqrt{25}$
4 — ? — 5

$\Delta = 25 - 16 = \boxed{9} \Rightarrow \frac{3}{9} = .\overline{3}$
$\Delta = 19 - 16 = \boxed{3}$ or $25 - 19 = 6$

b) $\sqrt{31} = 5.\overline{54}$

$\sqrt{25}$ — $\sqrt{31}$ — $\sqrt{36}$
5 — ? — 6

$\Delta = 36 - 25 = \boxed{11}$
$\Delta = 31 - 25 = \boxed{6}$
$\frac{6}{11} = 0.5454$

c) $\sqrt{47} = 6.846...$

$\sqrt{36}$ — $\sqrt{47}$ — $\sqrt{49}$
6 — ? — 7

$\Delta = 49 - 36 = 13$
$\Delta = 47 - 36 = 11$
$\frac{11}{13} = 0.84615...$

Method 3: Using your Calculator!

Your calculator is equipped with a magically button square root button, that can do all of this for you!

Practice:
Use your calculator to determine the answer to the following to a maximum of 4 decimal places if required.

a) $\sqrt{81}$ = 9
b) $\sqrt{(-16)}$ = Error!
c) $-\sqrt{9}$ = −3 oops!
d) $\sqrt{1}$ = 1
e) $\sqrt{(-16)}$ = Error!
f) $-\sqrt{9}$ = −3
g) $-\sqrt{45}$ = 6.7082...
h) $\sqrt{2}$ = 1.4142...
i) $-\sqrt{136}$ = −11.6619...
j) $\sqrt{576}$ = 24
k) $\sqrt{2116}$ = 46
l) $\sqrt{46225}$ = 215

Cubed Roots

The process is much the same for cubed root but granted more difficult. So, for cubed roots we only use our calculator!

Your calculator will be equipped with a button that looks like: $\sqrt[3]{\ }$, $\sqrt[x]{\ }$, $\sqrt[\square]{\ }$

Unlike square roots, a cubed root could negative (-) inside of the root sign.

Let us consider the following: √(-27)

This means that the product of ***three*** identical numbers produces a negative (-) value. We know from our integer rules the only way to make a negative (-) is by multiplying an odd number of negative (-) values...

(+3) x (+3) x (+3) = +29 → $(+3)^3$ = +27
(-3) x (-3) x (-3) = -27 → $(-3)^3$ = -27 Bingo!
(-3) x (+3) X (+3) = -27 → Bases are not the same, therefore no repeated multiplication, hence cannot be expressed as an exponent

However, you can have a negative outside (i.e. in front) of the square root. You can think of this as the square root being multiplied by negative one (-1)

Example:

a) $\sqrt[3]{(-8)}$ = -2 b) $\sqrt[3]{(-64)}$ = +4 c) $\sqrt[3]{(-125)}$ = -5

Practice:
Use your calculator to determine the answer to the following to a maximum of 4 decimal places if required.

a) $\sqrt[3]{120}$ = 4.9324... b) $\sqrt[3]{(-9)}$ = -2.0801... c) $-\sqrt[3]{27}$ = -3

d) $\sqrt[3]{1}$ = 1 e) $\sqrt[3]{(-216)}$ = -6 f) $-\sqrt[3]{(-64)}$ = +4

g) $\sqrt[3]{45}$ = 3.5569 h) $\sqrt[3]{20}$ = 2.7144... i) $-\sqrt[3]{343}$ = -7