Area & Volume Models

Area Models (2D)

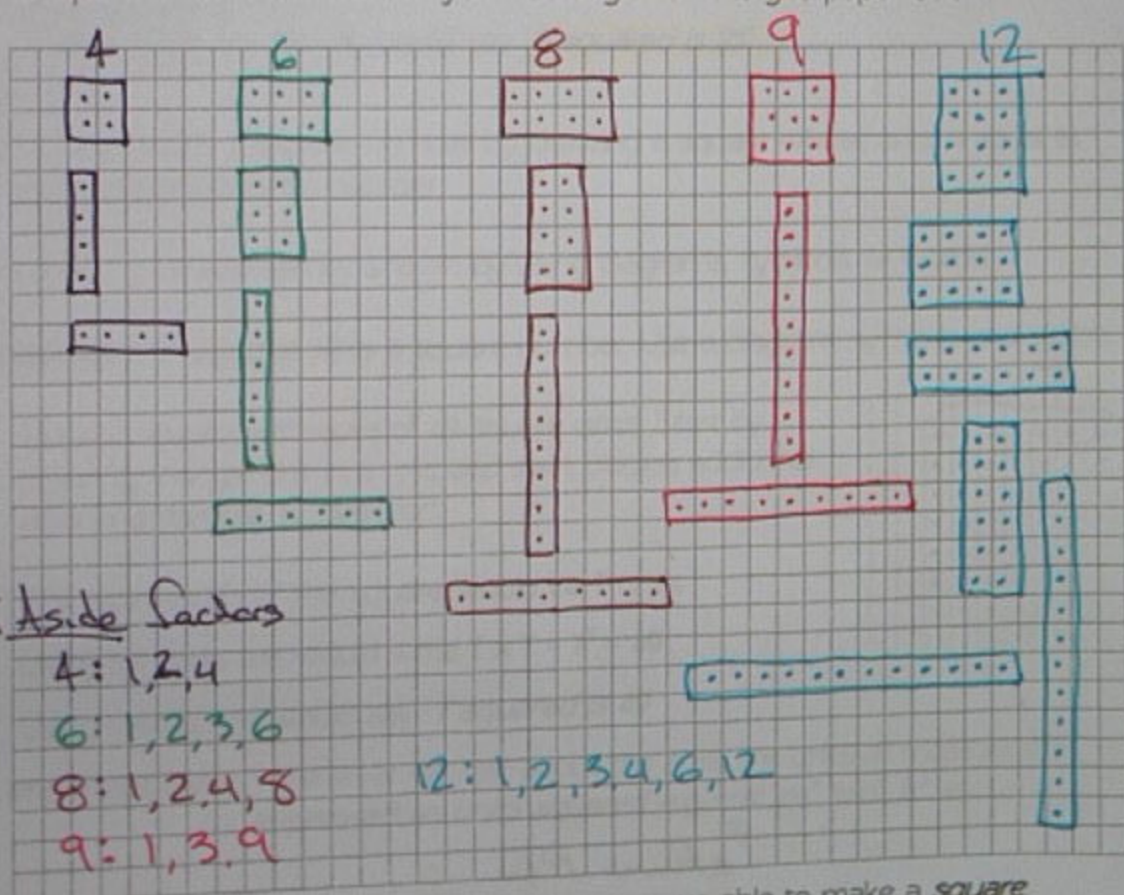
In our activity, what did you notice when creating perfect squares and rectangles?

⇒ The sides of a rectangle have different lengths and opposite sides are equal in length.
⇒ The sides of the perfect squares were all equal

Every square is a rectangle? True / False
Every rectangle is a square? True / False

Investigate

Using the grid below make as many different rectangles as you can with the following areas: **4** square units | **6** square units | **8** square units | **9** square units | **12** square units. Draw each of your rectangles on the grid paper below.

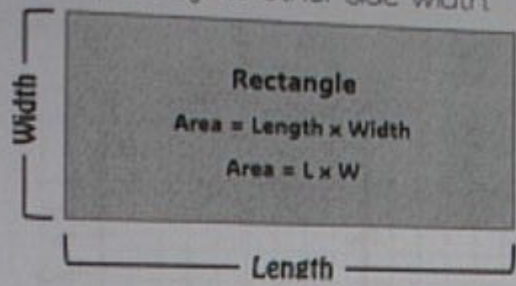


*Aside Factors
4: 1,2,4
6: 1,2,3,6
8: 1,2,4,8
9: 1,3,9
12: 1,2,3,4,6,12

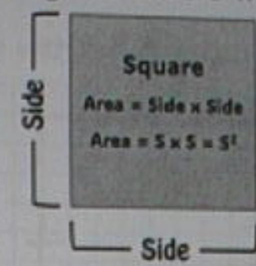
Circle which of the above areas you were able to make a ***square***

How is the side length of a square and rectangle related to its area?

The area is equal to its side length multiplied by its other side width.



But for a square, each side is the same length, therefore we get...



When we multiply a number by the same number, we call it *Squaring* the number.

Example:

⇒ The square of 4 is 16, we would say "*4 squared is 16*"

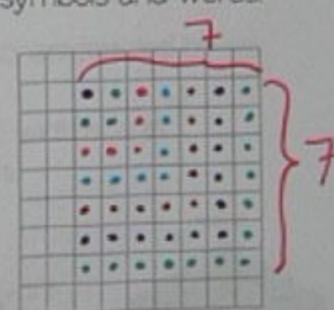
⇒ $5^2 = 5 \times 5 = 25$, we would say "*5 squared is 25*"

We can model a square number by drawing a square whose area is equal to the square of a given number.

Square numbers can be shown by using diagrams, symbols, or words!

Example: Show that 49 is a square number. Use a diagram, symbols and words.

Draw a square with area of 49 square units. Start by drawing a single dot in a grid and start placing additional dots in a reverse "L" direction.



The side length of the square would be 7 units

Using symbols we would write $7 \times 7 = 7^2 = 49$

Using words, we would say "*7 squared is 49*"

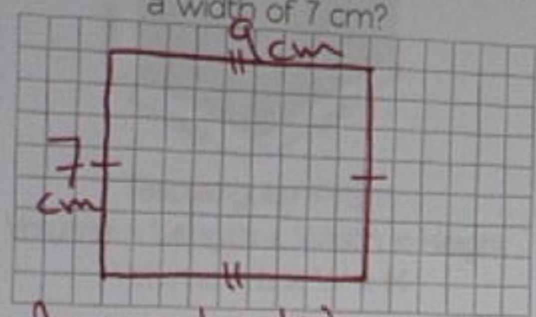
Important Note: Whenever we talk about the area, the units of measure are expressed as ***units squared*** (e.g. cm², m², km², ft², etc...)

49 units²

Practice:

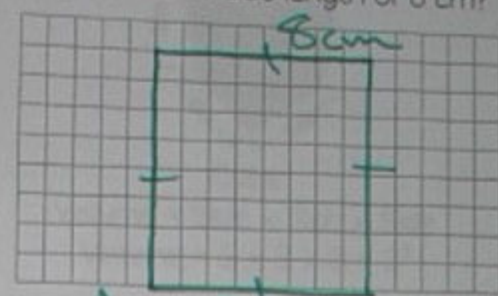
What is the area of the following shapes? Draw and label the shape on the grid provided. Which one has the largest area?

A rectangle with a length of 9 cm and a width of 7 cm?



Area = L × W
= 9 cm × 7 cm = ~~63~~
63 cm²

A square with a side length of 8 cm?



$A = s^2$
$= (8cm)^2 = 64 cm^2$

★ Larger

Volume Models (3D)

In our activity, you should have noticed distinct characteristics when constructing perfect cubes and rectangular prisms.

Perfect Cubes:

⇒ Perfect cubes are three-dimensional shapes where all edges (length, width, and height) are of equal length.
⇒ Each face of a perfect cube is a perfect square.

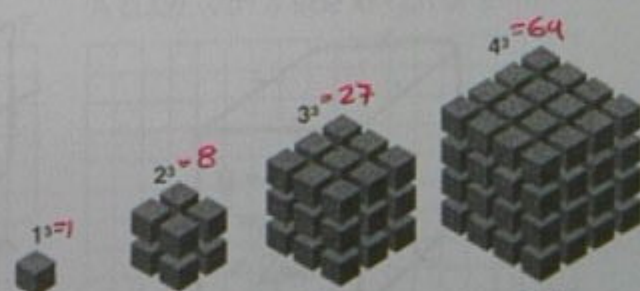
Rectangular Prisms:

⇒ Rectangular prisms are also three-dimensional shapes characterized by six faces (3-pairs), each of which is a rectangle.
⇒ Unlike cubes, rectangular prisms have different lengths, widths, and heights, allowing for varying dimensions along each axis.

Every cube is a rectangular prism? True / False

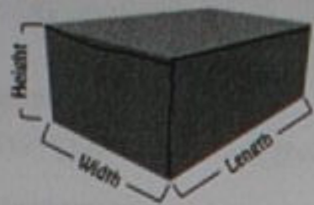
Every rectangular prism is a cube? True / False

How is the side length of a cube and rectangular prism related to its volume?



#Blocks 1 8 27 64

The volume is equal to its side length multiplied by its side width and side height



Rectangular Prism
Volume = Length x Width x Height
Volume = L x W x H

But for a Cube, each side is the same length, therefore we get...

Cube
Volume = Side x Side x Side
Volume = $S \times S \times S = S^3$

When we multiply a number by the same number twice, we call it ***Cubing*** the number.

Example:

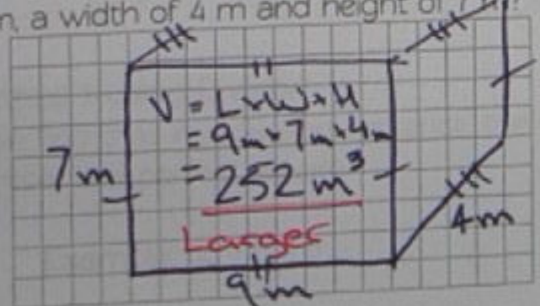
⇒ The cube of 4 is **64**, we would say "*4 cubed is 64*"

⇒ $5^3 = 5 \times 5 \times 5 = 125$, we would say "*5 cubed is 125*"

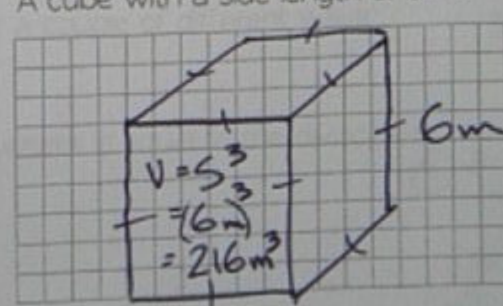
Important Note: Whenever we talk about the volume, the units of measure are expressed as ***units cubed*** (e.g. cm^3, m^3, km^3, ft^3, etc...)

Practice:
What is the volume of the following shapes? Draw and label the shape on the grid provided. Which one has the largest volume?

A rectangular prism with a length of 9 m, a width of 4 m and height of 7 m?



A cube with a side length of 6 m?



Perfect Squares and Square Roots

Recall a factor is a number that divides another number without leaving a remainder (i.e. no decimal). In other words, if you multiply a factor by another number, you get the original number as the result.

Investigate

The following chart shows the factors of each whole number from 1 to 30.

																							24						30
																							12						15
											12						18		20				8				28		10
											6				16		9		10				6				14		6
					6		8		10		4		14	15	8		6		5	21	22		4		26	27	7		5
			4		3		4	9	5		3		7	5	4		3		4	7	11		3	25	13	9	4		3
	2	3	2	5	2	7	2	3	2	11	2	13	2	3	2	17	2	19	2	3	2	23	2	5	2	3	2	29	2
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30

Which numbers have **only two** factors? What do you notice about these numbers?

⇒ Numbers that have only two factors are ***Prime Numbers***. They are divisible by only 1 and themselves.

Which numbers have an **odd** number of factors?

⇒ Since factors come in pairs, these are numbers that must have two of the same factors and, therefore must be ***perfect squares*** or **square numbers**!

⇒ The middle factor is the *Root number*!

M3 →

Prime Factorization = Even # of prime factors

36
3 12
3 4
2 2
(4) 2×2×3×3

Determining Perfect Squares

You can tell if a number is a perfect square a couple of different ways.

Method 1: Using Division:

If we find that in a division sentence, the quotient is equal to the divisor, the number is a perfect square.

Example: $36 \div 6 = 6$

144
12 12
6 2 2 6
2 3 2 3
(6) 2×2×2×2×3×3

Method 2: Using Factors:

If we find that a number has an ***ODD*** number of factors, then it is a square number.

Example: 25 has 3 factors: 1, 5, and 25.

Method 3: Using Prime Factorization:

If we find that a number has an ***EVEN*** number of prime factors, then it is a square number.

36 ^ 144 ^

Example: Prime factors of 25 are 5 and 5...so 5^2
Prime factors of 36 are 2, 2, 3 and 3...so 2^2 and 3^2

Method 4: Using Memory

You can memorize the list of common perfect squares or use your multiplication facts.

Perfect Squares to Memorize:				
	$0^2 = 0$	$4^2 = 16$	$8^2 = 64$	$11^2 = 121$
	$1^2 = 1$	$5^2 = 25$	$9^2 = 81$	$12^2 = 144$
	$2^2 = 4$	$6^2 = 36$	$10^2 = 100$	$13^2 = 169$
	$3^2 = 9$	$7^2 = 49$		

Square Roots

A square root ($\sqrt{\ }$) is a number which, when squared, results in a given number.

⇒ 3 is a square root of 9, so we write it as: $3 = \sqrt{9}$
⇒ The square root of 16 is 4, so we write is as: $\sqrt{16} = 4$.

FYI: square roots, and any root in general is called a *radical*! More on this in Grade 9 and up, but for now just understand the parts.

actually a cube root

square root would be $\sqrt[2]{\ }$

But we don't write the 2 so just $\sqrt{\ }$

radical sign

index $\sqrt[3]{64}$ radicand

Taking the square root of a number is the opposite operation of squaring a number, just like subtraction is the opposite of addition and division is of the opposite multiplication.

Squaring and taking the square root are ***inverse*** operations. They undo each other!

⇒ $3^2 = 9 \rightarrow \sqrt{9} = 3$
⇒ $4^2 = 16 \rightarrow \sqrt{16} = 4$

Taking a square root will work on any number, not just perfect squares.

What about Negative Integers?

Well, let's think about this...

$6^2 = 36$ & $(-6)^2 = 36$...therefore if we go backwards $\sqrt{36}$ = ???

But how do we know if the root was +6 or -6? Well, we don't, so to overcome this we use the following notion +/- or (±) to denote the answer could be either (+) or (-)

Therefore, the correct answer for $\sqrt{36} = \pm 6$

In some cases, the negative (-) answer maybe illogically, such as in a word problem; negative lengths, negative items, negative time...in these cases it is appropriate to express only the $(-4) \times (-4) = +14 \rightarrow (-4)^2 = +16$ positive (+) answer.

Caveat: This doesn't not apply in the reverse!

YOU CAN NOT TAKE A SQUARE ROOT OF A NEGATIVE NUMBER

Let us consider the following: $\sqrt{(-16)}$

This means that the product of ***two*** identical numbers produces a negative (-) value. But we know from our integer rules theh only way to make a negative (-) value is by multiplying a positive (+) number with a negative (-) number...therefore the numbers are not identical.

$(+4) \times (+4) = +16 \rightarrow (+4)^2 = +16$
$(-4) \times (-4) = +16 \rightarrow (-4)^2 = +16$
$(+4) \times (-4) = -16 \rightarrow$ Bases are not the same, therefore no repeated multiplication, hence cannot be expressed as an exponent

However, you can have a negative outside (i.e. in front) of the square root. You can think of this as the square root being multiplied by negative one (-1)

Example:

a) $-\sqrt{9}$ = -3

b) $-\sqrt{36}$ = -6

c) $-\sqrt{\cancel{81}}$ = Error!
(-81)

Non-Perfect Squares and Square Roots

What if the number is **NOT** a Perfect Square number?

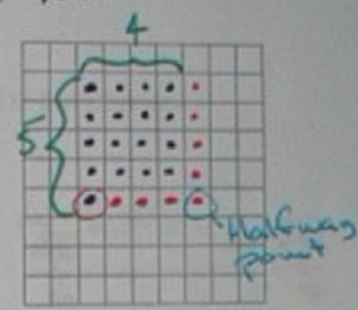
Estimating Square Roots

When dealing with non-perfect square numbers like 17 or 27, finding their square roots can be a bit challenging.

Method 1: Visually

A helpful way to estimate square roots is by visually constructing squares like we did previously by drawing dots.

Example: $\sqrt{17}$

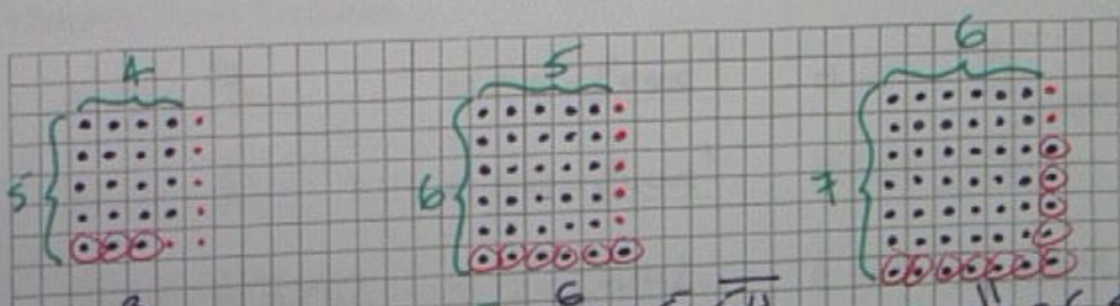


Step 1: Start by drawing 1 dot.
Step 2: Continue drawing dots in a backward "L" direction.
Step 3: Continue adding dots until you run out.
Step 4: Count the number of dots on each side of your array. The smaller value will tell you what whole number your value will start from.
Step 5: In your incomplete "L" count the number of dots ~~have~~ Needed to complete the 'L' 9
Step 6: Take the number dots you have divided by the total number for dots required to complete the "L". $\Rightarrow \frac{1}{9} = 0.11\bar{1}$
Step 7: Add this value as the decimal after your value determined in step 4. $4.\bar{1}$ actual value 4.12310....

99.7% (accuracy)

Practice: Estimate the square roots for the following using the visual method.

d) $\sqrt{19}$ = ________ e) $\sqrt{31}$ = ________ f) $\sqrt{47}$ = ________



$4 + \frac{3}{9} = 4.\bar{3}$ actual 4.359....

$5 + \frac{6}{11} = 5.\overline{54}$ actual 5.56

$6 + \frac{11}{13} = 6.8461...$ actual 6.85