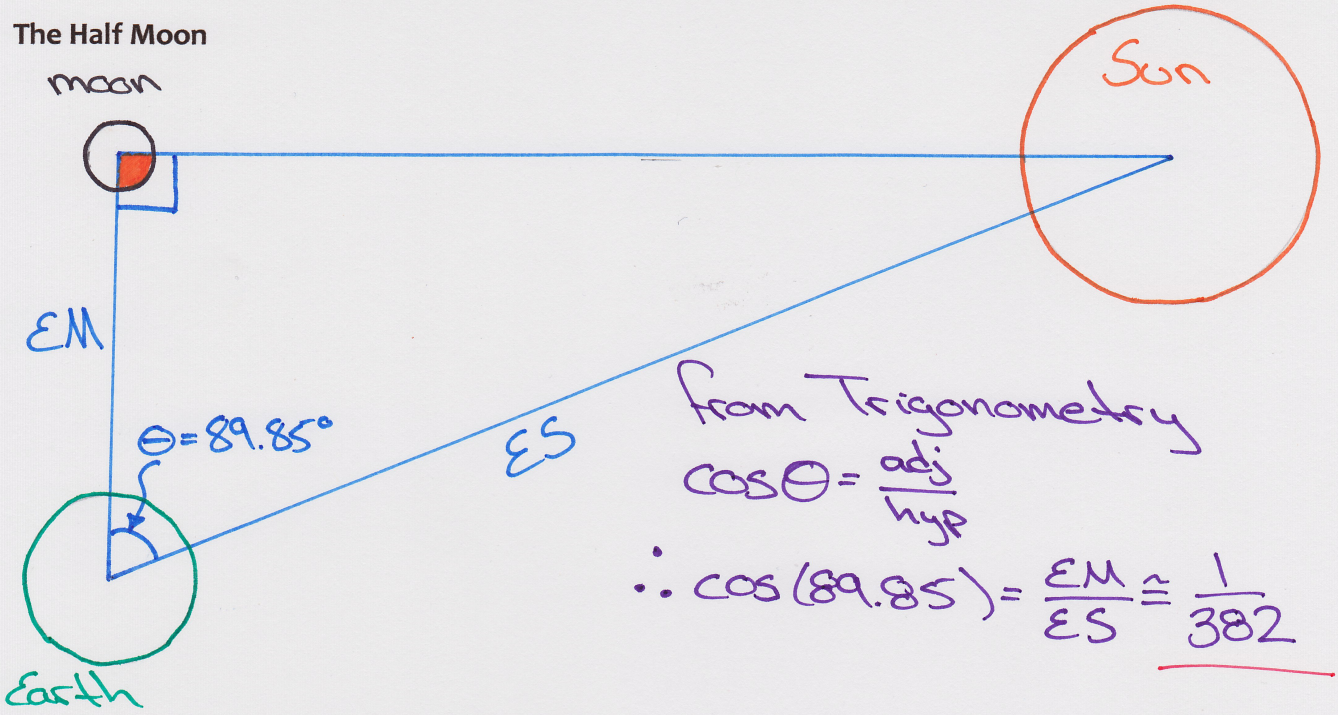
Aristarchus Calculation of the Size & Distances of the Sun & Moon (3rd Century BC)

The Half Moon

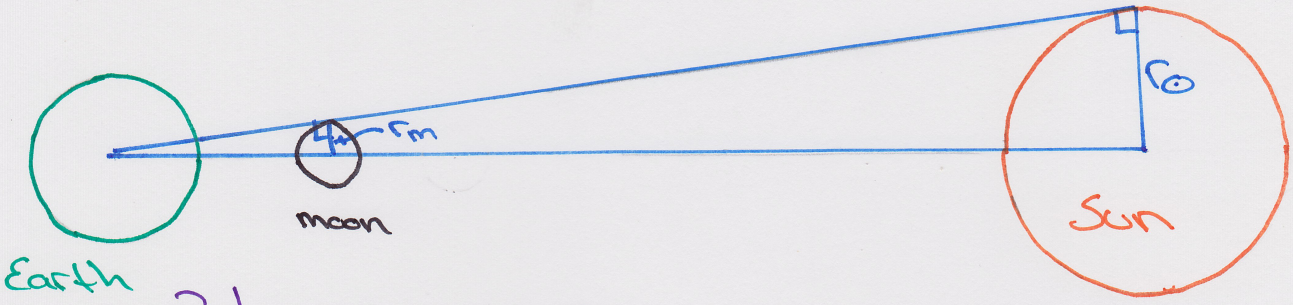


from Trigonometry

$\cos\theta = \frac{adj}{hyp}$

$\therefore \cos(89.85) = \frac{EM}{ES} \cong \frac{1}{382}$

Solar Eclipse

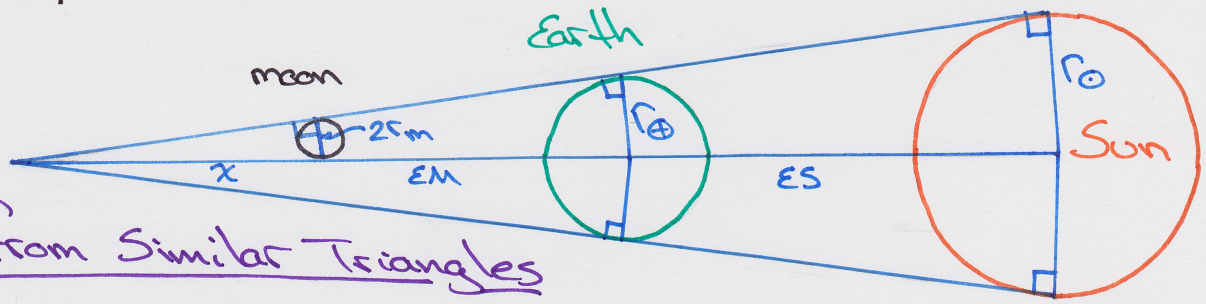


Ratio $\frac{\text{radius of Moon}}{\text{radius of Sun}} = \frac{\text{distance from Earth to Moon}}{\text{distance from Earth to Sun}}$

from Observation this ratio was determined to be $\frac{1}{19} = \frac{r_m}{r_s} = \frac{EM}{ES}$

Aristarchus Calculation of the Size & Distances of the Sun & Moon (3rd Century BC)

Lunar Eclipse



From Similar Triangles

$$\frac{x}{x+EM+ES} = \frac{2r_m}{r_\odot}$$

recall from Solar Eclipse $\Rightarrow \frac{1}{19} = \frac{r_m}{r_\odot} \therefore r_\odot = 19r_m$

$\therefore ES = 19EM$

Sub $\frac{x}{x+EM+19EM} = \frac{2r_m}{19r_m}$

$\frac{x}{x+20EM} = \frac{2}{19}$ solving for $x \Rightarrow 19x = 2x + 40EM$

$17x = 40EM$

$x = \frac{40EM}{17}$

Other Triangle

$$\frac{x}{x+EM} = \frac{2r_m}{r_\oplus}$$

$xr_\oplus = 2r_m x + 2r_m EM$ solving x

$$\left[\left(\frac{40EM}{17}\right)r_\oplus = 2r_m\left(\frac{40EM}{17}\right) + 2r_m EM\right] \times 17$$

$40EMr_\oplus = 80EMr_m + 34EMr_m$

$[40EMr_\oplus = 114EMr_m] \div 114EM$

$$\frac{40}{114}r_\oplus = r_m = \frac{20}{57}r_\oplus$$

Recall from Eratosthenes $r_\oplus = 6369\,km$

$\therefore r_m = \frac{20}{57}(6369\,km)$

$= 2235\,km$

Current estimate $r_m = 1740\,km$ → 77.9%

Aristarchus Calculation of the Size & Distances of the Sun & Moon (3rd Century BC)

Lunar Eclipse Continued

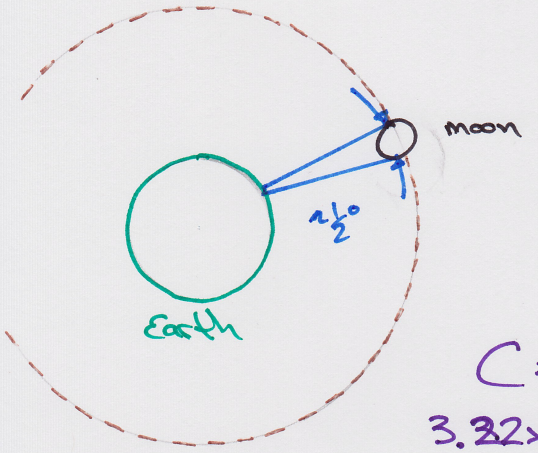
Now we can calculate the size of the Sun

Recall $\frac{1}{19} = \frac{r_m}{r_\odot}$ $\therefore r_\odot = 19 r_m = 19(2235\ km)$

$= \underline{42465\ km}$ → 6.1%

Present day value $r_\odot = 696000\ km$

Next we can determine the distance to the moon



From observation we estimate the 'angular diameter' of the moon to be 0.5°.

∴ it would take a total of 720 moon diameters to make a total of 360° (i.e. circle/orbit)

$C = 2\pi r$ where $C = 720 \times 2r_m$ (diameter)

$= 720 \times 2(2235\ km)$

$= 3.22 \times 10^6\ km$

$3.22 \times 10^6\ km = 2\pi r$

$\therefore r = \frac{3.22 \times 10^6\ km}{2\pi}$

$r = \underline{5.12 \times 10^5\ km}$

Present day value $3.84 \times 10^5\ km$ → 75%

Lastly, the distance to the Sun

Recall from the Half moon $\frac{EM}{ES} = \frac{1}{382}$

$\therefore ES = 382\,EM = 382(5.12 \times 10^5\ km)$

$= \underline{1.96 \times 10^8\ km}$

Present day value $1.5 \times 10^8\ km$ → 76.7%