

## Arithmetic Series

### What is a series?

- The sum of a sequence (the commas become + signs)

**Little Gauss:**  $1 + 2 + 3 + \dots \dots 97 + 98 + 99 + 100$  *he said look at first and last term*

Sum = 50(101)      Sum = 5050

**Lil Gauss' formula:**

$$S_n = \frac{n}{2}(a + t_n)$$

But this relies on knowing  $t_n$ :

$$S_n = \frac{n}{2}(a + a + (n - 1)d)$$

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

1) Find the indicated sum

a)  $3 + 8 + 13 \dots$  Find  $S_{162}$

$$a = 3 \quad d = 5 \quad n = 162 \quad S_{162} = \frac{162}{2}(2(3) + (162 - 1)5) \quad S_{162} = 81(6 + 805)$$

$$S_{162} = 65691$$

b)  $100 + 97 + 94 \dots$  Find  $S_{451}$

$$a = 100 \quad d = -3 \quad n = 451 \quad S_{451} = \frac{451}{2}(2(100) + (451 - 1)(-3))$$

$$S_{451} = 225.5(200 + -1350)$$

$$S_{451} = -259325$$

2) An arithmetic series has  $t_1 = 82$  and  $t_{10} = 130.6$ , find  $S_{30}$

We need 'd'  $a = 82, n = 10$  if  $t_n = 130.6$        $130.6 = 82 + (10-1)d$        $48.6 = 9d$        $d = 5.4$

$$\text{Now } n = 30, S_{30} = \frac{30}{2}(2(82) + (30 - 1)5.4) \quad S_{30} = 15(164 + 156.6) \quad S_{30} = 4809$$

4) Sum:  $7 + 14.2 + 21.4 + \dots + 331$

Can't use either formula without  $n$        $331 = 7 + (n-1)(7.2)$        $324 = 7.2(n-1)$        $45 = n-1$

$$n = 46 \quad S_{46} = \frac{46}{2}(2(7) + (46 - 1)7.2) \quad S_{46} = 23(14 + 324) \quad S_{46} = 7774$$

5) 145 people are at a Gala – if everyone shakes hands with everyone else once, how many handshakes will take place?

Person 1 shakes 144, person 2 shakes 143 ...  $d = -1$   $a = 144$   $n = 144$

$$S_{144} = \frac{144}{2}(2(144) + (144 - 1)(-1))$$

$$S_{144} = 10440 \text{ handshakes}$$

Assignment = worksheet

**Math 10 Arithmetic Series (Day 4 Worksheet)**

**1) Find the sum of the first 50 terms of**

a)  $1 + 5 + 9 + \dots$

b)  $2 + 5 + 8 + \dots$

c)  $10 + 8 + 6 + \dots$

d)  $0 - 2 - 4, \dots$

**2) Find the indicated sum**

a)  $7 + 23 + 39 + \dots \quad S_{46} = ?$

b)  $15 + 21.5 + 28 + \dots \quad S_{96} = ?$

c)  $99 + 85 + 71 + \dots \quad S_{99} = ?$

d)  $245 + 239 + 233 + \dots \quad S_{38} = ?$

e)  $94.5 + 103.2 + 111.9 + \dots \quad S_{444} = ?$

f)  $555 + 533 + 511 + \dots \quad S_{88} = ?$

3) A theatre has 30 seats in the first row, 32 seats in the second row, 34 seats in the third row and so on. How many seats are in the theatre if it contains 52 rows?

4) Use lil' Gauss' formula to sum the following

a)  $a = 6, t_{50} = 70$

b)  $a = 8, t_{44} = 112$

c)  $a = -12, t_{88} = 164$

5) Sum the following - these will require 2 steps

a)  $4 + 6 + 8 + \dots + 200$

b)  $3 + 7 + 11 + \dots + 479$

c)  $100 + 90 + 80 + \dots - 210$

d)  $-8 - 5 - 2 + \dots + 139$

e)  $18 + 12 + 6 + \dots - 216$

f)  $-7 - 11 - 15 - \dots - 171$

6) In an arithmetic series  $t_7 = 434$  and the common difference is  $-82$ , find  $S_{100}$ .

7) In an arithmetic series  $t_7 = 18$  and  $t_1 = -66$ , find  $S_{362}$ .