Lesson #7.0 – Factoring Review

Factoring a GCF — largest "value" common between all terms could be #'s or variables or Both!

This is always the first step in simplifying – taking out the common factor and returning the expression to its bracket form.

Example 1: Factor the following expression:

$4x^2 - 14$	$2x^2 + x$	$3x^3 - 12x$
factor out a 2	factor out x	factor out 3 & x
$2(2x^2 - 7)$	$x(2x+1)$	$3x\underbrace{(x^2-4)}$
		This can factor to.... learn this later!! :)

Factoring Trinomials

Factoring is the opposite of expanding ... so we will re-create the brackets. Looking back at expanding:

Foil or "Hills & Valley" Method!

$$(x + a)(x + b)$$
$$x^2 + bx + ax + ab$$
$$x^2 + (b + a)x + ab$$

Notice the middle term is the sum of $\boldsymbol{a}$ and $\boldsymbol{b}$ and the last term is a product of $\boldsymbol{a}$ and $\boldsymbol{b}$.

$$Ax^2 + Bx + C$$

☆ Use this phrase when factoring trinomials: ***Find 2 numbers that multiply to C and add to B.***

Example 2: Factor each trinomial

$x^2 + 4x - 21$	$x^2 + 7x + 10$
multiple $\Rightarrow 7 \times (-3) = -21$	multiply $\Rightarrow 5 \times 2 = 10$
add $\Rightarrow 7 + (-3) = 4$	add $\Rightarrow 5 + 2 = 10$
$(x+7)(x-3)$	$(x+5)(x+2)$

Factoring by Decomposition
Suppose your trinomial has a number in front of the x^2 (and is NOT a GCF) then this number needs to be part of the expansion and part of the brackets

$2x^2 + 9x - 5$	**Step 1:** multiply first and last (2 x 5)
	Step 2: find 2 numbers that multiply to this and add to middle term (in this case 10 x 1)
$2x^2 + 10x - 1x - 5$	**Step 3:** decompose the middle term using these 2 numbers
$2x(x + 5) - 1(x + 5)$	**Step 4:** factor GCF from first 2 and last 2 terms
$(2x - 1)(x + 5)$	**Step 5:** factor new GCF and state answer

Example 3: Factor each trinomial

$2x^2 + 7x + 6$

$2\times 6 = 12$
$3\times 4 = 12$
$3+4=7$

$(2x^2+4x)+(3x+6)$
$2x(x+2)+3(x+2)$
$(2x+3)(x+2)$

$4x^2 + 11x - 3$

$4\times(-3) = -12$
$+12\times(-1) = -12$
$+12+(-1) = +11$

$(4x^2+12x)-(x-3)$
$4x(x+3)-1(x+3)$
$(4x-1)(x+3)$

Factoring Special Cases

Perfect Squares result when $Ax^2 + Bx + C$ factors to $\rightarrow (px + q)^2$ where A and C are perfect square numbers ($1^2 =$ **1**, $2^2 =$ **4**, $3^2 =$ **9**, $4^2 =$ **16**, $5^2 =$ **25**, $6^2 =$ **36**, $7^2 =$ **49**, $8^2 =$ **64**, $9^2 =$ **81**, $10^2 =$ **100**, $11^2 =$ **121**, $12^2 =$ **144**, etc….). The middle term B is the equal to $2 \times \sqrt{A} \times \sqrt{C}$.

$\sqrt{4}=2$ $\sqrt{49}=7$
$4x^2 - 28x + 49$
$(2x-7)^2$

$\sqrt{9}=3$ $\sqrt{1}=1$
$9x^2 + 6x + 1$
$(3x+1)^2$

$\sqrt{16}=4$ $\sqrt{9}=3$
$16x^2 - 24xy + 9y^2$
$(4x-3)^2$

Difference of Squares occurs when A and C are perfect square numbers but value of C is negative resulting in the middle term, B to become zero (0) and $Ax^2 + 0x - C$ factors to $\rightarrow (px - q)^2$

$x^2 - 16$ $\sqrt{16}=4$
$(x+4)(x-4)$
Expand $\Rightarrow$ $x^2 \underbrace{+4x-4x}_{\text{cancel out!}} -16$

$\sqrt{4}=2$ $\sqrt{9}=3$
$4x^2 - 9$
$(2x+3)(2x-3)$

$\sqrt{4}=2$ $\sqrt{49}=7$
$-4x^2 + 49$
$-(4x^2-49)$
$-(2x+7)(2x-7)$

Example 4: Factor each expression fully

$-3x^2 + 9x - 6$

*Pull GCF 1st is possible!

$-3(x^2-3x+2)$
$-3(x-2)(x-1)$

$2x^2 - 12x + 18$

$2(x^2-6x+9)$
$2(x-3)(x-3)$
$2(x-3)^2$

Quadratic Equation: $ax^2 + bx + c = 0$

Many quadratic equations can be solved by factoring. The ***Zero Product Property*** states that if $\boldsymbol{ab = 0}$, then $\boldsymbol{a = 0}$ or $\boldsymbol{b = 0}$, or **both**. Therefore, the roots of a quadratic equation occur when the product of the factors equal to zero.

Example 4: Solve each equation, then verify the solution.

$x^2 - 2x - 8 = 0$

$(x+2)(x-4)=0$
if $x=-2$
$(-2+2)(-2-4)=0$
$0(-6)=0$
$0=0$!!
or if $x=4$
$(4+2)(4-4)=0$
$6(0)=0$
$0=0$!!

$2x^2 + 18 = 12x$

$2x^2-12x+18=0$
$2(x^2-6x+9)=0$
$2(x-3)(x-3)=0$
$2(x-3)^2=0$
$x=3$
$2(3)^2+18=12(3)$
$18+18=36$
$36=36$ ✓!!

$2x^2 = 4x \Rightarrow 2x^2-4x$

$2x(x-2)$
if $x=2$
or
$x=0$
$2(2)^2=4(2)$
$2(4)=4(2)$
$8=8$
!!

Example 4: A football is kicked perfectly vertically. The approximate height of the football, h metres, and t seconds is modeled by the formula:

$$h = 1 + 20t - 5t^2$$

a) Determine the height of the football after 2 seconds.

$t=2$
$h=1+20(2)-5(2)^2$
$h=1+40-5(4)$
$h=41-20=\underline{21\text{ m}}$

b) When is the football 16 m high?

$h=16$
$16=1+20t-5t^2$
$5t^2-20t+15=0$
$5(t^2-4t+3)=0$
$5(t-3)(t-1)=0$
$t=\underline{3_s}$ and $t=\underline{1_s}$

Practice Questions: Factoring Review Worksheet