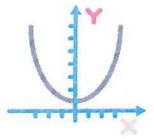


FOUNDATIONS OF MATH 11

Chapter 7 – Quadratic Functions and Equations



Lesson #7.0 – Factoring Review

Factoring a GCF

This is always the first step in simplifying – taking out the common factor and returning the expression to its bracket form.

— largest "value" common between all terms could be #s or variables or Both!

Example 1: Factor the following expression:

$$4x^2 - 14$$

factor out a 2

$$2(2x^2 - 7)$$

$$2x^2 + x$$

factor out x

$$x(2x + 1)$$

$$3x^3 - 12x$$

factor out 3ix

$$3x(x^2 - 4)$$

This can factor to... learn this later!!

Factoring Trinomials

Factoring is the opposite of expanding ... so we will re-create the brackets. Looking back at expanding:

$$\begin{aligned} (x + a)(x + b) \\ x^2 + bx + ax + ab \\ x^2 + (b + a)x + ab \end{aligned}$$

FOIL or "Hills : Valley" Method!

Notice the middle term is the sum of **a** and **b** and the last term is a product of **a** and **b**.

$$Ax^2 + Bx + C$$

★ Use this phrase when factoring trinomials: **Find 2 numbers that multiply to C and add to B.**

Example 2: Factor each trinomial

$$x^2 + 4x - 21$$

multiply $\Rightarrow 7 \times (-3) = -21$
add $\Rightarrow 7 + (-3) = 4$

$$(x + 7)(x - 3)$$

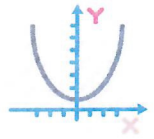
$$x^2 + 7x + 10$$

multiply $\Rightarrow 5 \times 2 = 10$
add $\Rightarrow 5 + 2 = 7$

$$(x + 5)(x + 2)$$

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Factoring by Decomposition

Suppose your trinomial has a number in front of the x^2 (and is NOT a GCF) then this number needs to be part of the expansion and part of the brackets

- | | |
|------------------------|--|
| $2x^2 + 9x - 5$ | Step 1: multiply first and last (2×5) |
| $2x^2 + 10x - 1x - 5$ | Step 2: find 2 numbers that multiply to this and add to middle term (in this case 10×1) |
| $2x(x + 5) - 1(x + 5)$ | Step 3: decompose the middle term using these 2 numbers |
| $(2x - 1)(x + 5)$ | Step 4: factor GCF from first 2 and last 2 terms |
| | Step 5: factor new GCF and state answer |

Example 3: Factor each trinomial

$2 \times 6 = 12$ $3 \times 4 = 12$ $3 + 4 = 7$	$2x^2 + 7x + 6$ $(2x^2 + 4x) + (3x + 6)$ $2x(x + 2) + 3(x + 2)$ $(2x + 3)(x + 2)$	$4x^2 + 11x - 3$ $(4x^2 + 12x) - (x - 3)$ $4x(x + 3) - 1(x + 3)$ $(4x - 1)(x + 3)$	$4 \times (-3) = -12$ $+12 \times (-1) = -12$ $+12 + (-1) = +11$
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Factoring Special Cases

Perfect Squares result when $Ax^2 + Bx + C$ factors to $(px + q)^2$ where A and C are perfect square numbers ($1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16, 5^2 = 25, 6^2 = 36, 7^2 = 49, 8^2 = 64, 9^2 = 81, 10^2 = 100, 11^2 = 121, 12^2 = 144$, etc....). The middle term B is the equal to $2 \times \sqrt{A} \times \sqrt{C}$.

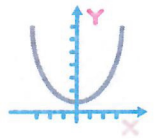
$\sqrt{4} = 2$	$\sqrt{49} = 7$	$\sqrt{9} = 3$	$\sqrt{1} = 1$	$\sqrt{16} = 4$	$\sqrt{9} = 3$
$4x^2 - 28x + 49$		$9x^2 + 6x + 1$		$16x^2 - 24xy + 9y^2$	
$(2x - 7)^2$		$(3x + 1)^2$		$(4x - 3)^2$	

Difference of Squares occurs when A and C are perfect square numbers but value of C is negative resulting in the middle term, B to become zero (0) and $Ax^2 + 0x - C$ factors to $(px - q)^2$

$\sqrt{16} = 4$	$\sqrt{4} = 2$	$\sqrt{9} = 3$	$\sqrt{4} = 2$	$\sqrt{49} = 7$
$x^2 - 16$	$4x^2 - 9$		$-4x^2 + 49$	
$(x + 4)(x - 4)$	$(2x + 3)(2x - 3)$		$-(4x^2 - 49)$	$-(2x + 7)(2x - 7)$
Expand $\Rightarrow x^2 + 4x - 4x - 16$ <div style="margin-left: 100px;"> $\underbrace{\quad\quad}_{\text{cancel out!}}$ </div>				

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Example 4: Factor each expression fully

$-3x^2 + 9x - 6$ **Pull GCF! is possible!* $2x^2 - 12x + 18$

$-3(x^2 - 3x + 2)$ $2(x^2 - 6x + 9)$
 $-3(x-2)(x-1)$ $2(x-3)(x-3)$
 $2(x-3)^2$

Quadratic Equation: $ax^2 + bx + c = 0$

Many quadratic equations can be solved by factoring. The **Zero Product Property** states that if $ab = 0$, then $a = 0$ or $b = 0$, or **both**. Therefore, the roots of a quadratic equation occur when the product of the factors equal to zero.

Example 4: Solve each equation, then verify the solution.

$x^2 - 2x - 8 = 0$ $(x+2)(x-4) = 0$ if $x = -2$ $(-2+2)(-2-4) = 0$ $0(-6) = 0$ $0 = 0$!! or if $x = 4$ $(4+2)(4-4) = 0$ $6(0) = 0$ $0 = 0$!!	$2x^2 + 18 = 12x$ $2x^2 - 12x + 18 = 0$ $2(x^2 - 6x + 9) = 0$ $2(x-3)(x-3) = 0$ $2(x-3)^2 = 0$ $x = 3$ $2(3)^2 + 18 = 12(3)$	$2x^2 = 4x \Rightarrow 2x^2 - 4x$ $2x(x-2)$ if $x = 2$ or $x = 0$ $2(2)^2 = 4(2)$ $2(4) = 4(2)$ $8 = 8$!! !!
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Example 4: A football is kicked perfectly vertically. The approximate height of the football, h metres, and t seconds is modeled by the formula:

$$h = 1 + 20t - 5t^2$$

a) Determine the height of the football after 2 seconds.

$t = 2$
 $h = 1 + 20(2) - 5(2)^2$
 $h = 1 + 40 - 5(4)$

$\rightarrow h = 41 - 20 = \underline{\underline{21 \text{ m}}}$

b) When is the football 16 m high?

$h = 16$
 $16 = 1 + 20t - 5t^2$

$\rightarrow 5t^2 - 20t + 15 = 0$
 $5(t^2 - 4t + 3) = 0$
 $5(t-3)(t-1) = 0$
 $t = \underline{\underline{3}}$ and $t = \underline{\underline{1}}$

Practice Questions: Factoring Review Worksheet