FOM 11 **6.4 Optimization Problems I: Creating the Model**

Linear inequalities can be used to solve **optimization problems**, problems in which we find the greatest or least value of functions. The method used to solve such problems is called **linear programming**, and consists of two parts:

1. An **objective function** tells us the quantity we want to maximize or minimize.
2. The system of constraints consists of linear inequalities whose region is called the **feasible solution** with area called the **feasible region**.

Example 1: A company makes motorcycles and bicycles. A restricted work area limits the numbers of vehicles that can be made in one day: no more than 10 motorcycles can be made, no more than 15 bicycles can be made, and no more than 20 vehicles of both kinds can be made. If the profit is \$25 for a motorcycle and \$50 for a bicycle, what should be the daily rate of production of both vehicles to maximize the profits?

total

Step 1: Define the variables that affect the quantity to be optimized and state any restrictions.

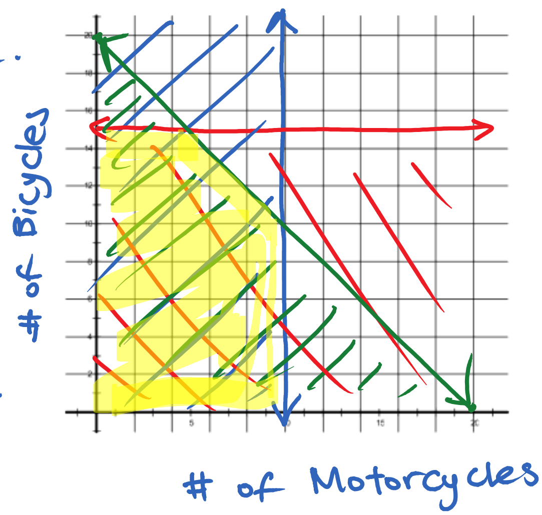
Let x = # of motorcycles

y = # of bicycles

$x + y$ have to be whole numbers (W).

Step 2: Identify the quantity that must be optimized.

Profit must be maximized.



Step 3: Write an objective function.

$P = 25x + 50y$

Step 4: Write a system of linear inequalities to describe all the constraints of the problem and graph the feasible solution.

No more than 10 motorcycles.

$x \leq 10$

No more than 15 bicycles

$y \leq 15$

No more than 20 all together

$x + y \leq 20$

$y = -x + 20$

Example 2: Fred is planning an exercise program where he wants to run and swim every week. He doesn't want to spend more than 12 hours a week exercising and he wants to burn at least 1600 calories a week. Running burns 200 calories an hour and swimming burns 400 calories an hour. Running costs \$1 an hour while swimming costs \$2 an hour. How many hours should he spend at each sport to keep his costs at a minimum?

Step 1: Define the variables that affect the quantity to be optimized and state any restrictions.

Let x = # hours of running

y = # hours of swimming

x & y have to be whole numbers.

Step 2: Identify the quantity that must be optimized.

Cost must be minimized.

Step 3: Write an objective function.

$$C = 1x + 2y$$

Step 4: Write a system of linear inequalities to describe all the constraints of the problem and graph the feasible solution.

No more than 12 hours

$$x + y \leq 12$$

x-int
plug $y=0$
$x + 0 = 12$
$x = 12$ $(12, 0)$

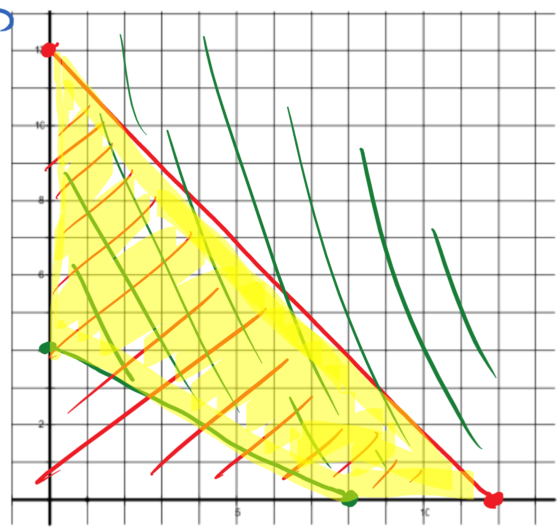
y-int
plug $x=0$
$0 + y = 12$
$y = 12$ $(0, 12)$

Burns @ least 1600 calories

$$200x + 400y \geq 1600$$

x-int
$y=0$
$200x + 400(0) = 1600$
$200x = 1600$
$x = 8$
$(8, 0)$

y-int.
$x=0$
$200(0) + 400y = 1600$
$400y = 1600$
$y = 4$



# of hours of swimming

# of hours of running

Assignment: $\boxed{\text{pg. 330 \#1-7 (odd)}}$