We often hear statements such as:

> "80% of dentists recommend this toothpaste with results accurate
> within 2 percentage points, 19 times out of 20".

When we interpret this result, we need to consider the **confidence interval**, the **margin of error**, and the **confidence level.**

A confidence interval is the interval in which the true value is estimated to lie, with a stated degree of probability. The confidence interval may be expressed using ± notation, such as 54.0% ± 3.5%, or ranging from 50.5% to 57.5%.

$$80\% \pm 2\% \quad \text{OR} \quad \text{from } 78\% \text{ to } 82\%$$

The **margin of error** is the possible difference between the estimate of the value you are trying to determine and the true value for the population. (Usually expressed as a plus or minus percent, such as ± 5%)

$$\text{margin of error} \quad \pm 2\%$$

The **confidence level** is the likelihood that the result for the "true" population lies within the range of the confidence interval.

$$\frac{19}{20} \quad \text{or} \quad 95\%$$

Example 1: A survey of 320 users of the skateboard park indicates that 40% of them would like the parks board to extend the evening use of the facility. This survey is considered accurate to within 5.4%, 19 times out of 20.

a. Calculate the range of people that want to extend the evening use of the facility.

Margin of error $\pm 5.4\%$

Confidence Interval $40\% \pm 5.4\% \Rightarrow 34.6\%$ to 45.4%

$0.346 \times 320 = 110.72$

$0.454 \times 320 = 145.28$

The range of people that want to extend use is 111 to 145 people.
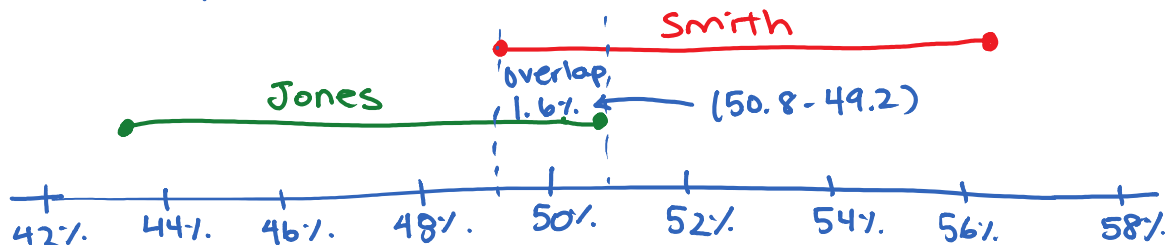
b. Determine the accuracy of the results.

Confidence level $\frac{19}{20}$ or 95%.

The results are 95% correct.

Example 2: A poll was conducted to ask voters the following question: If an election were held today, whom would you vote for? The results indicated that 53% would vote for Smith and 47% would vote for Jones. The results were stated as being accurate within 3.8 percent points, 19 times out of 20. Who will win the election?

Smith: $53\% \pm 3.8 \Rightarrow 49.2\%$ to 56.8%.

Jones: $47\% \pm 3.8 \Rightarrow 43.2\%$ to 50.8%.

Smith

Jones

overlap, 1.6% $\leftarrow$ (50.8 - 49.2)

42% 44% 46% 48% 50% 52% 54% 56% 58%

Smith is most likely to win, but there is an overlap of 1.6% where there is a possibility that Jones could win.

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Example 3: Polling organizations in Canada frequently survey samples of the population to gauge voter preference prior to elections. People are asked:

1. "If an election were held today, which party would you vote for?" If they say they don't know, then they are asked:
2. "Which party are you leaning toward voting for?"

The results of three different polls taken during the first week of November, 2010, are shown below. The results of each poll are considered accurate 19 times out of 20.

Polling Organization & Data	Conservative (%)	Liberal (%)	NDP (%)	Bloc Quebecois (%)	Green Party (%)	Undecided (%)
Ekos	29	29	19	9	11	12.6
sample size, 1815 margin of error, ±2.3%						
Nanos	37	32	15	11	5	19.2
sample size, 844 margin of error, ±3.4%						
Ipsos	35	29	12	11	12	n.a.
sample size, 1000 margin of error, ±3.1%						

source: http://www.sfu.ca/~aheard/elections/polls.html

a. How does the sample size used in the poll affect the margin of error in the reported results?

b. Compare the confidence intervals for the Liberal Party for each of the three polls. How does the sample size used in the poll affect the confidence interval?

Example 4: To meet regulation standards, baseballs must have a mass from 142.0 g to 149.0 g. A manufacturing company has set its production equipment to create baseballs that have a mean mass of 145.0 g. To ensure that the production equipment continues to operate as expected, the quality control engineer takes a random sample of baseballs each day and measures their mass to determine the mean mass. If the mean mass of the random sample is 144.7 g to 145.3 g, then the production equipment is running correctly. If the mean mass of the sample is outside the acceptable level, the production equipment is shut down and adjusted. The quality control engineer refers to the chart shown below when conducting random sampling.

Confidence Level	Sample Size Needed
99%	110
95%	65
90%	45

a. What is the confidence interval and margin of error the engineer is using for quality control tests?

b. What is the relationship between confidence level and sample size?

c. After making adjustments in equipment, the quality control engineer decided that the mean mass of baseballs must lie in the range 144.2 g to 146.4 g.

    i. What is the mean mass and the margin of error?

    ii. Will the new baseballs meet regulation standards?

Assignment: Pg. 274 #3, 4, 6-9