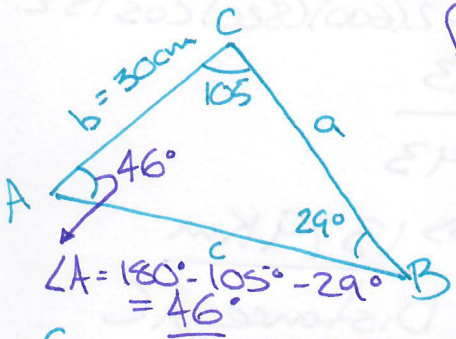
Lesson #4.2 – Proving and Applying the Sine and Cosine Laws in Obtuse Triangles

The **Sine Law** works for obtuse triangle as well as acute triangles

Example 1: Solve ΔABC, given ∠B = 29°, ∠C = 105°, and b = 30 cm.

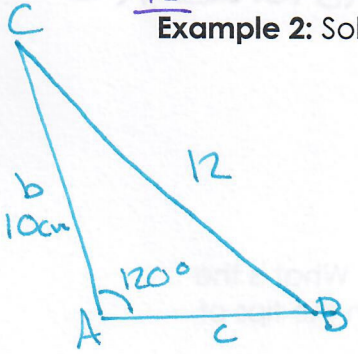


$\angle A = 180° - 105° - 29° = 46°$

find a: $\frac{a}{\sin 46} = \frac{30}{\sin 29} \Rightarrow a = \frac{30 \sin 46}{\sin 29} = 44.5\text{ cm}$

find b: $\frac{c}{\sin 105} = \frac{30}{\sin 29} \Rightarrow c = \frac{30 \sin 105}{\sin 29} = 59.8\text{ cm}$

Example 2: Solve ΔABC, given ∠A = 120°, a = 12, and b = 10 cm, find ∠B.



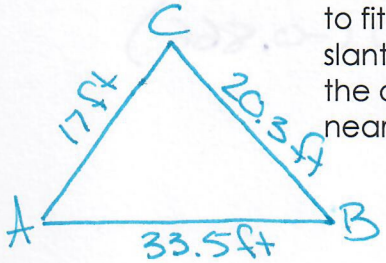
$\frac{\sin B}{10} = \frac{\sin 120°}{12}$

$\sin B = \frac{10 \sin 120}{12}$

$B = \sin^{-1}(0.722) = 46.2°$

The **Cosine Law** works for obtuse triangle as well as acute triangles

Example 3: The roof of a house consists of two slanted sections, as shown. A roofing cap is being made to fit the crown of the roof, where the two slanted sections meet. Determine the measure of the angle elevation for each roof section, to the nearest tenth of a degree.



find ∠A

$a^2 = b^2 + c^2 - 2bc \cdot \cos A$

$20.3^2 = 17^2 + 33.5^2 - 2(17 \times 33.5) \cdot \cos A$

$412 = 289 + 1122 - 1139 \cos A$

$412 = 1411 - 1139 \cos A$

$-999 = -1139 \cos A$

$A = \cos^{-1}\left(\frac{999}{1139}\right)$

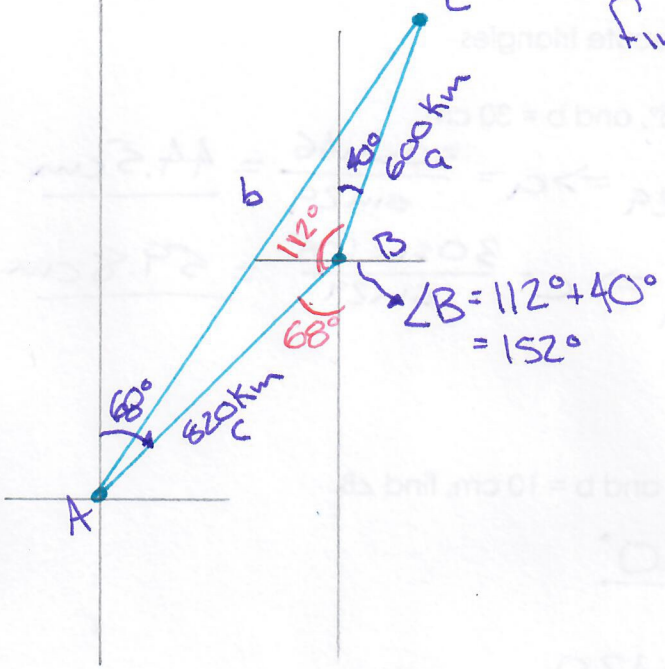
$A = 28.7°$

find ∠B:

$\frac{\sin B}{17} = \frac{\sin 28.7}{20.3}$

$\angle B = 23.7°$

Example 4: A plane flies 820 km from A to B at a bearing of N86°E. Then it flies 600 km from B to C with a bearing of N40°E. Find the distance from C to A.



$\angle B = 112° + 40° = 152°$

Find AC = b:

$b^2 = a^2 + c^2 - 2ac \cdot \cos B$

$b^2 = 600^2 + 820^2 - 2(600)(820)\cos 152$

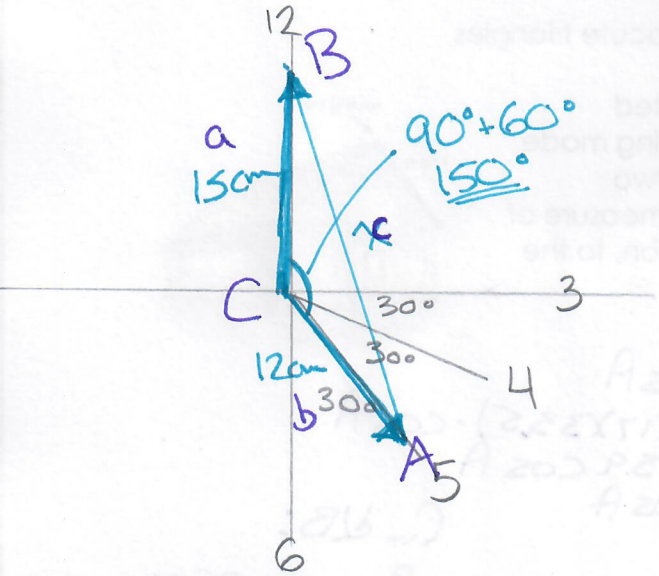
$b^2 = 1901220.43$

$b = \sqrt{1901220.43}$

$b = 1378.8 \Rightarrow 1379\,Km$

Distance AC

Example 5: A clock has two hands that are 12 cm and 15 cm long. What is the distance to the nearest tenth of a centimetre, between the tips of the hands at 5 p.m.?



Find c:

$c^2 = a^2 + b^2 - 2ab \cdot \cos C$

$c^2 = 15^2 + 12^2 - 2(15)(12)\cos 150°$

$c^2 = 225 + 144 - 360(-0.866)$

$c^2 = 369 + 311.7$

$c^2 = 680.8$

$c = \sqrt{680.8}$

$c = 26\,cm$

Practice Questions: Page 170, #'s 1, 2, 4, 6-9, 12-14