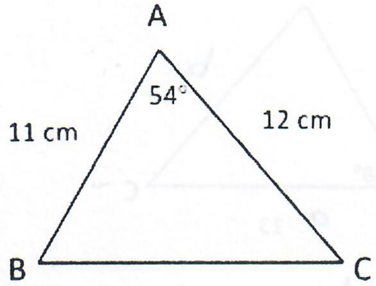
Lesson #3.2 – Proving and Applying the Cosine Law

The Sine Law cannot always help you determine the unknown sides and angles in an acute triangle, as seen the following example:



Therefore, another relationship is needed to solve these situations.

The **Cosine Law** states:

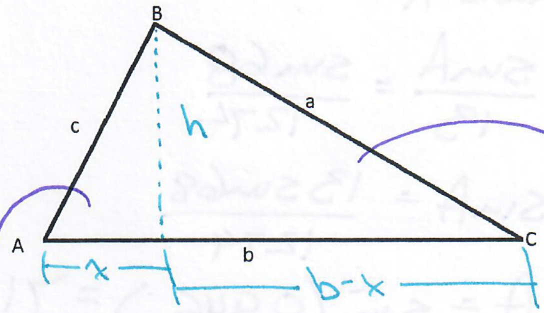
$$a^2 = b^2 + c^2 - 2bc \cdot cosA$$

or

$$b^2 = a^2 + c^2 - 2ac \cdot cosB$$

or

$$c^2 = a^2 + b^2 - 2ab \cdot cosC$$



Proof:

$c^2 = h^2 + x^2$

$x = c\cos A$

$a^2 = h^2 + (b-x)^2$

$a^2 = h^2 + b^2 - 2bx + x^2$

$a^2 = h^2 + x^2 + b^2 - 2bx$

$a^2 = c^2 + b^2 - 2bx$

$\therefore a^2 = c^2 + b^2 - 2bc \cdot \cos A$

similarly:

$b^2 = a^2 + c^2 - 2ac \cdot \cos B$

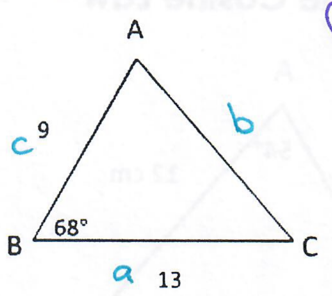
$\vdots$

$c^2 = a^2 + b^2 - 2ab \cdot \cos C$

***To use the Cosine Law, you need either:**

- Two (2) sides and one (1) contained angle.
- All three (3) sides

Example 1: Solve for unknown side and angles.



find b: $b^2 = a^2 + c^2 - 2ac \cdot \cos B$

$b^2 = 13^2 + 9^2 - 2(13)(9) \cdot \cos 68$

$b^2 = 169 + 81 - 234 \cdot \cos 68$

$\sqrt{b^2} = \sqrt{162.34}$

$b = \underline{12.74}$

find A

$\frac{\sin A}{13} = \frac{\sin 68}{12.74}$

$\sin A = \frac{13 \sin 68}{12.74}$

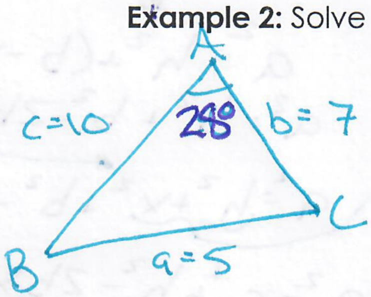
$A = \sin^{-1}(0.946\ldots) = 71°$

find C:

$180° - 68° - 71°$

$= \underline{41°}$

Example 2: Solve $\triangle ABC$ where a = 5, b = 7 & c = 10.



Find A:

$a^2 = b^2 + c^2 - 2bc \cdot \cos A$

$5^2 = 7^2 + 10^2 - 2(7)(10) \cdot \cos A$

$25 = 49 + 100 - 140 \cos A$

$25 = 149 - 140 \cos A$

$-149 \quad -149$

$\frac{-124}{-140} = \frac{-140 \cos A}{-140}$

$0.8857 = \cos A$

$A = \cos^{-1}(0.8857)$

$A = 27.66 = \underline{\underline{28°}}$

find B:

$\frac{\sin B}{7} = \frac{\sin 28}{5}$

$B = \sin^{-1}(0.657\ldots)$

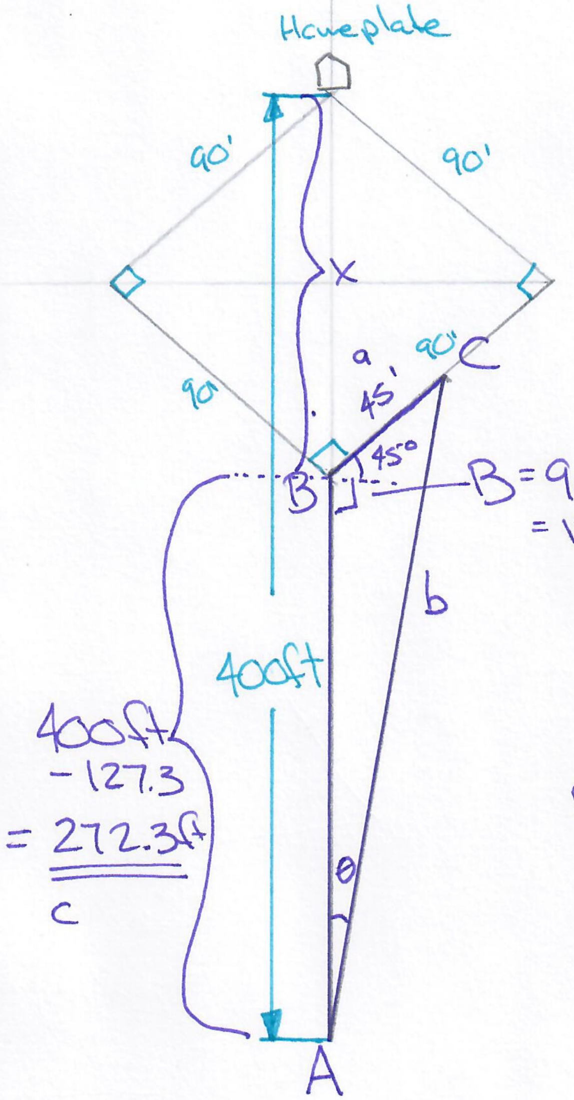
$B = \underline{\underline{41°}}$

Find C:

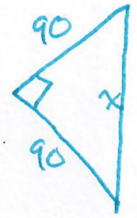
$180 - 28 - 41$

$= \underline{\underline{111°}}$

Example 3: The distance from Homeplate to center field at a baseball stadium is 400 ft. What is the angle at center field between the lines of sight to the short stop (half-way between 2nd and 3rd base), and home plate? Note: the distance between each adjacent base is 90 ft.



find x: $x^2 = a^2 + b^2$



$x^2 = 90^2 + 90^2$

$x^2 = 16200$

$x = \sqrt{16200} = \underline{127.3\text{ ft}}$

find b:

$b^2 = a^2 + c^2 - 2ac \cdot \cos B$

$b^2 = 45^2 + 272^2 - 2(45)(272) \cdot \cos 135$

$b^2 = 93749.68..$

$b = \sqrt{93749.68..}$

$b = \underline{306.19\text{ ft}}$

find θ:

$$\frac{\sin\theta}{45} = \frac{\sin 135}{306.2}$$

$\theta = \sin^{-1}(0.1039...)$

$\theta = 5.965 \Rightarrow \underline{6°}$

Practice Questions: Page 137, #'s 2-5, 6ac, 7b, 8, 9, 13