## Factors

Express 28 as a product of two natural numbers.

$\Rightarrow 28 = 2 \times 14$

$\Rightarrow 28 = 4 \times 7$

$\Rightarrow 28 = 1 \times 28$

We can also say 28 can be exactly divided by: 1, 2, 4, 7, 14, 28

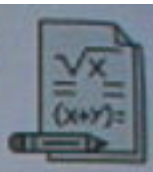
**A FACTOR is a number that, when multiplied with another number, produces a given product.**

Therefore, the factors of 28 are: 1, 2, 4, 7, 14, 28

What are the factors for 36? 1, 2, 3, 4, 6, 9, 12, 18, 36

### Divisibility Test:

1. A number is divisible by 2 if it's *Last* digit is Even
   Example: 2, 4, 6, 8, .... 20, 22, ... 98, 100, ...
2. A number is divisible by 3 if the *Sum* of its digits is also divisible by 3
   Example: 3, 6, 9, 18, 21, 27, 36, 42, 51, 102, 111, 132, 195, ...
3. A number is divisible by 5 if its Last digit is either 0 or 5
   Example: 5, 10, 15, 20, 50, 55, 85, 105, 140, ...
4. A number is divisible by 10 if its Last digit is 0
   Example: 10, 20, 50, 70, 110, 140, 160, 170, ...



## Multiples:

28 is a multiple of 4 and 7.

The first few multiples of 4 are:

⇒ 4 x 1 = 4
⇒ 4 x 2 = 8
⇒ 4 x 3 = 12
⇒ 4 x 4 = 16
⇒ 4 x 5 = 20

The first few multiples of 7 are:

⇒ 7 x 1 = 7
⇒ 7 x 2 = 14
⇒ 7 x 3 = 21
⇒ 7 x 4 = 28
⇒ 7 x 5 = 35

Relationships of 28 = 4 x 7

⇒ 4 is a factor of 28.
⇒ 28 is a multiple of 4.
⇒ 7 is a factor of 28.
⇒ 28 is a multiple of 7.

## Prime Factorization:

Consider the number 60. It is a ***composite*** number as it can be written as a product of 2 or more other numbers (60 = 6 x 10, 2 x 30, 3 x 20, 4 x 15, etc.)

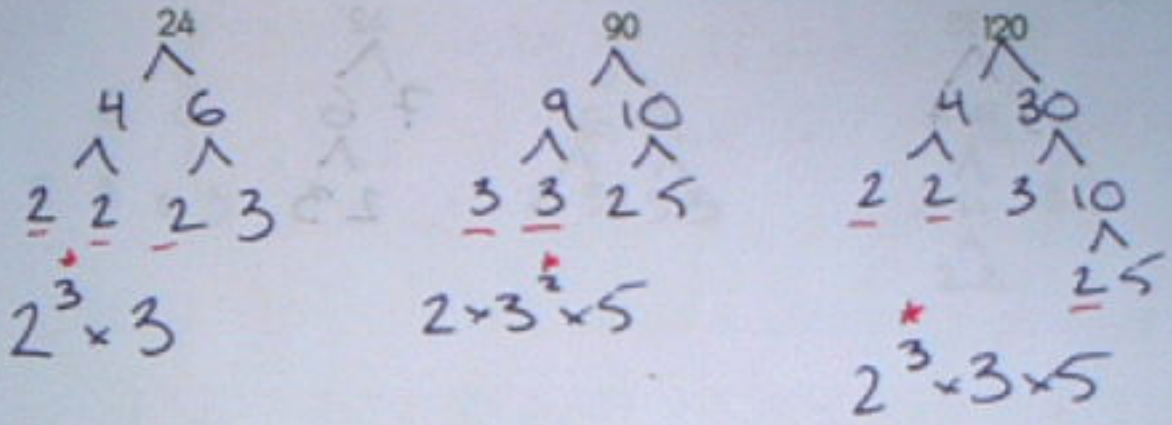
If we break down a composite number and its factors until there are no more factors, we end up with a number made ***only from prime numbers*** (60 = 2 x 2 x 3 x 5).*

⇒ When a number is written as a ***Product of Prime Numbers*** we call this a **Prime Factorization**

**Recall:** Prime numbers are special numbers that have exactly two factors: 1 and the number itself (ie. they are only divisible by 1 and themselves).

**Common Prime #'s:** 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, ...

**Example:** Using a factor tree determine the Prime Factors of the following numbers? Express your answer in exponential form.




## The Greatest Common Factor (GCF)

Let's find all the factors 18 and 24 as follows:

⇒ 18: 1, 2, 3, 6, 9, 18

⇒ 24: 1, 2, 3, 4, 6, 8, 12, 24

What are the common factors between 18 and 24? 1, 2, 3, 6

What would be the **greatest** (i.e. biggest/largest) common factor? GCF (18, 24) 6

The Greatest Common Factor is the largest factor that is the same (i.e. common) between a set of numbers.

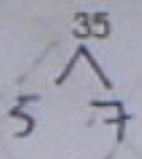
OR

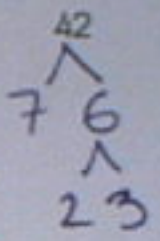
The largest number that can be divided evenly into all the numbers.

This method can be tricky and take a lot of time. Alternatively, we can use ***Prime Factorization*** to help us find the GCF.

The **GCF** can be found by identifying ***ALL*** the Prime Factors ***common*** between the set of numbers and multiplying them together.

**Example:** Find the Prime Factors for the following pairs of numbers using a factor tree, then find the GCF



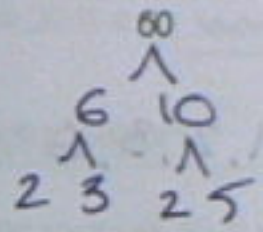



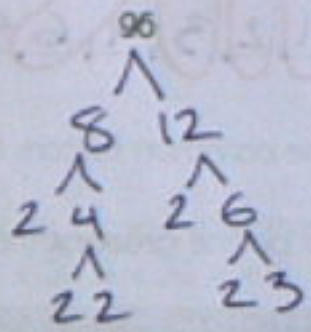

The Prime Factors of 35 and 42 are:

⇒ 35: 5, 7

⇒ 42: 2, 3, 7

⇒ GCF = 7



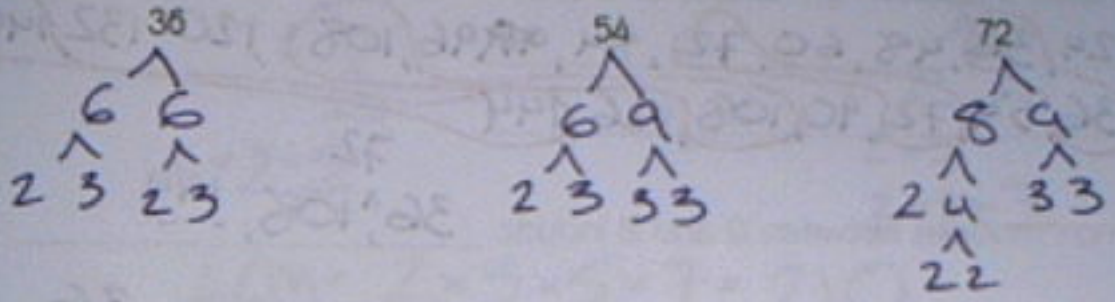

The Prime Factors of 60 and 90 are:

⇒ 60: $2^2 \times 3 \times 5$ or $2 \times 2 \times 3 \times 5$

⇒ 96: $2^5 \times 3$ or $2 \times 2 \times 2 \times 2 \times 2 \times 3$

⇒ GCF = $2 \times 2 \times 3 = 12$

**Example:** Find the Prime Factors for the following set of numbers using a factor tree, then find the GCF



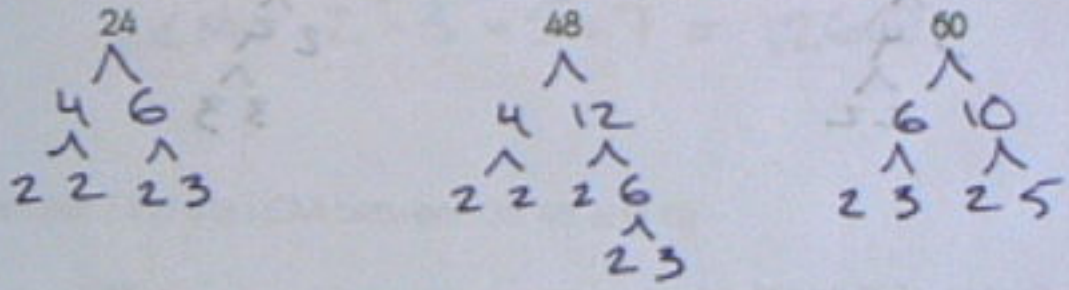

The Prime Factors of 36, 54 and 72 are:

⇒ 36: $2 \times 2 \times 3 \times 3$

⇒ 54: $2 \times 3 \times 3 \times 3$

⇒ 72: $2 \times 2 \times 2 \times 3 \times 3$

⇒ GCF = $2 \times 3 \times 3 = 18$




The Prime Factors of 24, 48 and 60 are:

⇒ 24: $2 \times 2 \times 2 \times 3$

⇒ 48: $2 \times 2 \times 2 \times 2 \times 3$

⇒ 60: $2 \times 2 \times 3 \times 5$

⇒ GCF = $2 \times 2 \times 3 = 12$

## Lesson 4 Lowest Common Multiple (LCM)

Let us consider the following multiples of 12 and 18:

⇒ 12: 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144

⇒ 18: 36, 54, 72, 90, 108, 126, 144

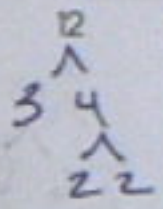
The common multiples between 12 and 18 include: 36, 72, 108, 144

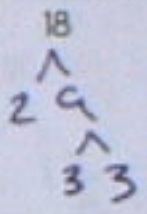
What is the **lowest** common multiple between 12 and 18? LCM (12, 18) = 36

**The Lowest Common Multiple is the smallest number that is a multiple of each of the given numbers.**

Again, this method can be tricky and take a lot of time. Alternatively, we can use ***Prime Factorization*** to help us find the LCM.

**Example:** Find the Prime Factors for 12 and 18




The Prime Factors of 12 and 18 are:

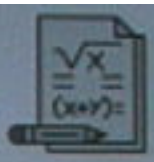
⇒ 12: $2 \times 2 \times 3$ Exponential Form: $2^2 \times 3$

⇒ 18: $2 \times 3 \times 3$ Exponential Form: $2 \times 3^2$

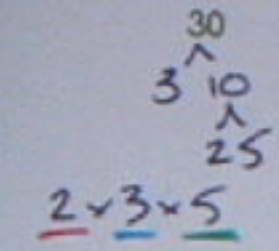
The **LCM** can be found by identifying ***ALL*** the prime factors of each number in ***exponential form*** and then multiplying the ***highest powers*** for each of the prime factors.

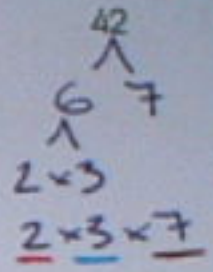
⇒ Prime Factors: $2^2 \times 3$ // $2 \times 3^2$

⇒ LCM = $2^2 \times 3^2 = 4 \times 9 = 36$



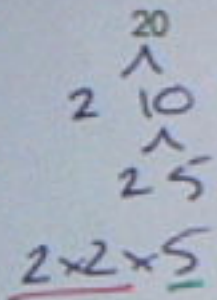
**Example:** Find the LCM between 30 and 42

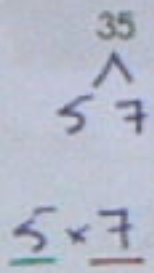


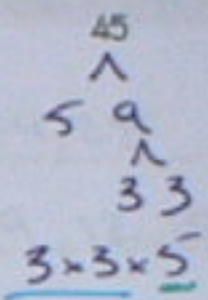




$LCM = 2 \times 3 \times 5 \times 7 = 210$

**Example:** Find the LCM between 20, 35, and 45.

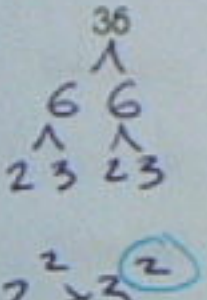


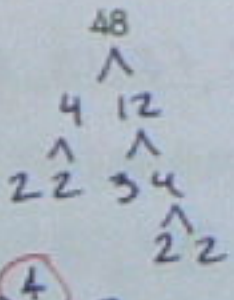



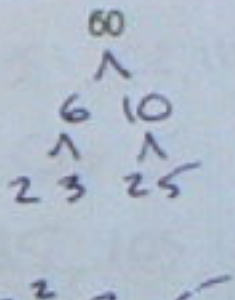




$LCM = 2^2 \times 3^2 \times 5 \times 7 = 1260$

**Example:** Find the LCM between 36, 48, and 60.




$LCM = 2^4 \times 3^2 \times 5 = 720$